

Autumn
Scheme of learning
Year 6

**White
Rose
Maths**

[#MathsEveryoneCan](#)

The White Rose Maths schemes of learning

Teaching for mastery

Our research-based schemes of learning are designed to support a mastery approach to teaching and learning and are consistent with the aims and objectives of the National Curriculum.

Putting number first

Our schemes have number at their heart. A significant amount of time is spent reinforcing number in order to build competency and ensure children can confidently access the rest of the curriculum.

Depth before breadth

Our easy-to-follow schemes support teachers to stay within the required key stage so that children acquire depth of knowledge in each topic. Opportunities to revisit previously learned skills are built into later blocks.

Working together

Children can progress through the schemes as a whole group, encouraging students of all abilities to support each other in their learning.

Fluency, reasoning and problem solving

Our schemes develop all three key areas of the National Curriculum, giving children the knowledge and skills they need to become confident mathematicians.

Concrete – Pictorial – Abstract (CPA)

Research shows that all children, when introduced to a new concept, should have the opportunity to build competency by following the CPA approach. This features throughout our schemes of learning.

Concrete

Children should have the opportunity to work with physical objects/concrete resources, in order to bring the maths to life and to build understanding of what they are doing.



Pictorial

Alongside concrete resources, children should work with pictorial representations, making links to the concrete. Visualising a problem in this way can help children to reason and to solve problems.



Abstract

With the support of both the concrete and pictorial representations, children can develop their understanding of abstract methods.

An abstract representation of the addition problem 5 + 7. The equation $5 + 7$ is written inside a yellow rectangular box.

If you have questions about this approach and would like to consider appropriate CPD, please visit www.whiterosemaths.com to find a course that's right for you.

Teacher guidance

Every block in our schemes of learning is broken down into manageable small steps, and we provide comprehensive teacher guidance for each one. Here are the features included in each step.

Notes and guidance that provide an overview of the content of the step and ideas for teaching, along with advice on progression and where a topic fits within the curriculum.

Things to look out for, which highlights common mistakes, misconceptions and areas that may require additional support.

Year 5 | Autumn Term | Block 1 – Place Value | Step 1

Roman numerals to 1,000

Notes and guidance

In Year 4, children learned about Roman numerals to 100. In this small step, they explore Roman numerals to 1,000, and the symbols D (500) and M (1,000) are introduced. Children explore further the similarities and differences between the Roman number system and our number system, learning that the Roman system does not have a zero and does not use placeholders. Children use their knowledge of M and D to recognise years using Roman numerals. Asking children to write the date in Roman numerals is one way to reinforce the concept daily.

Things to look out for

- Children may mix up which letter stands for which number.
- Children may add the individual values together instead of interpreting the values based on their position, for example interpreting CD as 600 instead of 400
- It is often more difficult to convert numbers that require large strings of Roman numerals.
- Children may think that numbers such as 990 can be written as XM instead of CMXC.

Key questions

- What patterns can you see in the Roman number system?
- What rules do we use when converting numbers to Roman numerals?
- What letters are used in the Roman number system? What does each letter represent?
- How do you know what order to write the letters when using Roman numerals?
- What is the same and what is different about representing the number “five hundred and three” in the Roman number system and in our number system?

Possible sentence stems

- The letter ____ represents the number ____
- I know ____ is greater than ____ because ____

National Curriculum links

- Read Roman numerals to 1,000 (M) and recognise years written in Roman numerals

© White Rose Maths 2022

Key questions that can be posed to children to develop their mathematical vocabulary and reasoning skills, digging deeper into the content.

Possible sentence stems to further support children’s mathematical language and to develop their reasoning skills.

National Curriculum links to indicate the objective(s) being addressed by the step.

Teacher guidance

A **Key learning** section, which provides plenty of exemplar questions that can be used when teaching the topic.

Year 2 | Autumn Term | Block 1 - Place Value | Step 1

Numbers to 20

Key learning

- Complete the number tracks.
 - 0 1 2
 - 10 11 12
 - 7 8 13
- What numbers are shown?
 - Give your answers in numerals and words.
- What numbers are shown?
 - Give your answers in numerals and words.
- Use words to complete the sentences.
 - The number after four is _____
 - The number before eight is _____
 - The number after nine is _____

© White Rose Maths 2022

Activity symbols that indicate an idea can be explored practically

Reasoning and problem-solving activities and questions that can be used in class to provide further challenge and to encourage deeper understanding of each topic.

Year 3 | Autumn Term | Block 1 - Place Value | Step 4

Hundreds

Reasoning and problem solving

Dora: I am going to count in 100s from zero.

Write two numbers that Dora will say.

any two multiples of 100

No

Mo is counting in hundreds.

... 8 hundred, 9 hundred, 10 hundred

How should Mo have said the last number?

Mo should have said 1 thousand, 10 hundreds is equal to 1 thousand.

Tiny: Dora will say the number 160

Is Tiny correct? How do you know?

Balloons come in bags of 10

Rosie has 300 balloons.

Rosie has 30 bags of balloons.

© White Rose Maths 2022

Answers provided where appropriate

Activities and symbols

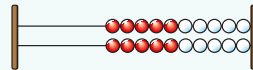
Key Stage 1 activities

Key Stage 1 includes more hands-on activities alongside questions.

An activity to be led by the teacher



Use a Rekenrek in the ready position.

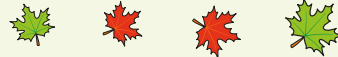


Ask children to show a number on their Rekenrek.

An outside activity or one that uses resources from nature



Find some seeds and leaves to represent Autumn.



Ask children to sort the objects in three different ways and then compare their answers with a partner.

An activity introduced by a reading from an appropriate fiction or non-fiction book



Read *The Button Box* by M Reid.

Give children a selection of buttons and ask them to sort the buttons in as many different ways as they can.

Encourage them to think about size, shape, colour and number of holes.

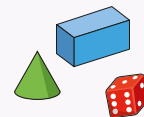


An investigation



Give children a selection of 3D shapes.

Ask children to sort the objects into two groups and then challenge a partner to say how the objects have been sorted.



Key Stage 1 and 2 symbols

The following symbols are used to indicate:



concrete resources might be useful to help answer the question



a bar model might be useful to help answer the question



drawing a picture might help children to answer the question



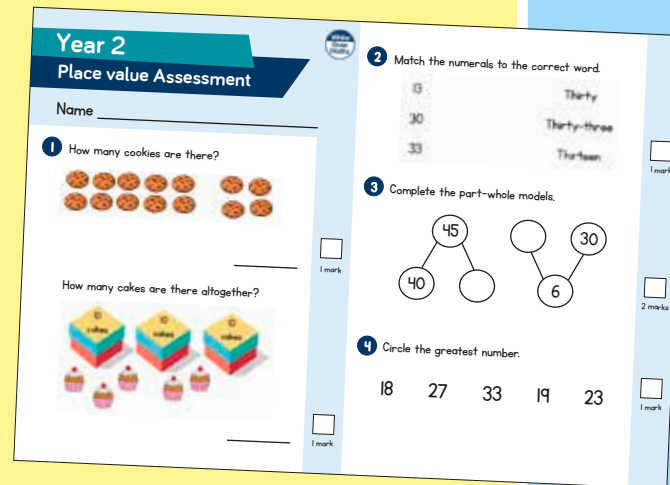
children talk about and compare their answers and reasoning



a question that should really make children think. The question may be structured differently or require a different approach from others and/or tease out common misconceptions.


Free supporting materials


End-of-block assessments to check progress and identify gaps in knowledge and understanding.



Year 2
Place value Assessment

Name _____

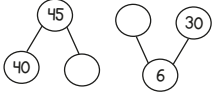
1 How many cookies are there?

_____ 1 mark

How many cakes are there altogether?

_____ 1 mark

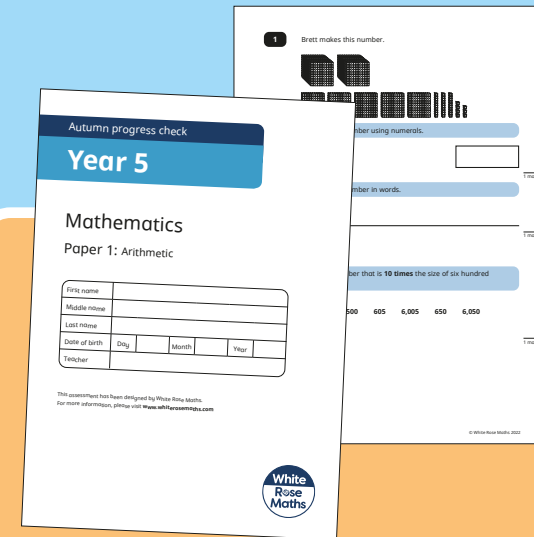
2 Match the numerals to the correct word.

13	Thirty
30	Thirty-three
33	Thirteen

_____ 1 mark

3 Complete the part-whole models.

_____ 2 marks

4 Circle the greatest number.
18 27 33 19 23
_____ 1 mark



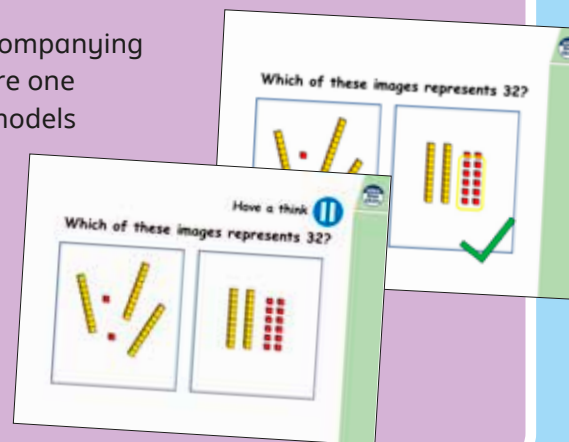
Autumn progress check
Year 5
Mathematics
Paper 1: Arithmetic


First name			
Middle name			
Last name			
Date of birth	Day	Month	Year
Teacher			


This assessment has been designed by White Rose Maths. For more information, please visit www.white-rose-maths.com

White Rose Maths

Each small step has an accompanying **home learning video** where one of our team of specialists models the learning in the step. These can also be used to support students who are absent or who need to catch up content from earlier blocks or years.



Which of these images represents 32?


Have a think
Which of these images represents 32?


End-of-term assessments for a more summative view of where children are succeeding and where they may need more support.

Free supporting materials

Primary Progression – Place Value						
	Year 1	Year 2	Year 3	Year 4	Year 5	Year 6
Place Value: Counting	<ul style="list-style-type: none"> count to and across 100, forwards and backwards, beginning with 0 or 1, or from any given number Count numbers to 100 in numerals; count in multiples of twos, fives and tens <p>Autumn 1 Autumn 4 Spring 2 Summer 4</p>	<ul style="list-style-type: none"> count in steps of 2, 3, and 5 from 0, and in tens from any number, forward and backward <p>Autumn 1</p>	<ul style="list-style-type: none"> count from 0 in multiples of 4, 8, 50 and 100, find 10 or 100 more or less than a given number <p>Autumn 1 Autumn 3</p>	<ul style="list-style-type: none"> count in multiples of 6, 7, 9, 25 and 1000 count backwards through zero to include negative numbers <p>Autumn 1 Autumn 4</p>	<ul style="list-style-type: none"> count forwards or backwards in steps of powers of 10 for any given number up to 1 000 000 count forwards and backwards with positive and negative whole numbers, including through zero <p>Autumn 1</p>	

National Curriculum progression to indicate how the schemes of learning fit into the wider picture and how learning progresses within and between year groups.

Skill: Add three 1-digit numbers

Year: 2

When adding three 1-digit numbers, children should be encouraged to look for number bonds to 10 or doubles to add the numbers more efficiently.

This supports children in their understanding of commutativity.

Manipulatives that highlight number bonds to 10 are effective when adding three 1-digit numbers.

$7 + 6 + 3 = 16$

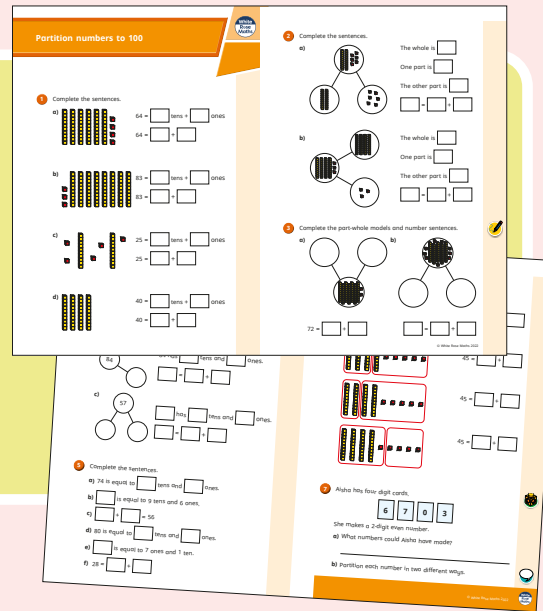
Calculation policies that show how key approaches develop from Year 1 to Year 6.

Ready to Progress – Number Facts Year 3			
	3NF-1	3NF-2	3NF-3
RTP Criteria	Secure fluency in addition and subtraction facts that bridge 10, through continued practice.	Recall multiplication facts, and corresponding division facts, in the 10, 5, 2, 4 and 8 multiplication tables, and recognise products in these multiplication tables as multiples of the corresponding number.	Apply place-value knowledge to know additive and multiplicative number facts (scaling facts by 10).
White Rose Maths Small Steps	Autumn 2 Addition and Subtraction <ul style="list-style-type: none"> Add 3-digit and 1-digit numbers - crossing 10 Subtract a 1-digit number from a 3-digit number - crossing 10 Add 3-digit and 2-digit numbers - crossing 100 Subtract a 2-digit number from a 3-digit number - crossing 100 	Autumn 3 Multiplication and Division <ul style="list-style-type: none"> 2 times-table 5 times-table Divide by 2 Divide by 5 Divide by 10 Multiply by 4 Divide by 4 The 4 times-table Multiply by 8 Divide by 8 The 8 times-table 	Spring 1 Multiplication and Division <ul style="list-style-type: none"> Related calculations Scaling Spring 4 Measurement: Length and Perimeter <ul style="list-style-type: none"> Equivalent lengths (m and cm) Equivalent lengths (mm and cm)

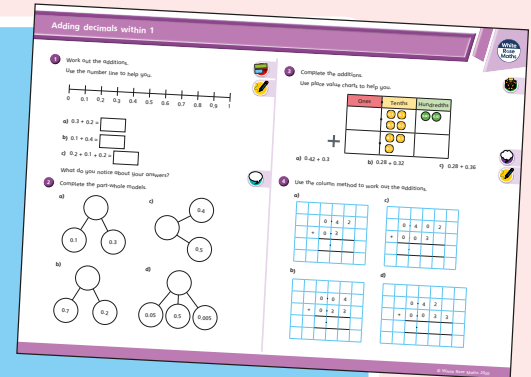
Ready to progress mapping that shows how the schemes of learning link to curriculum prioritisation.

Premium supporting materials

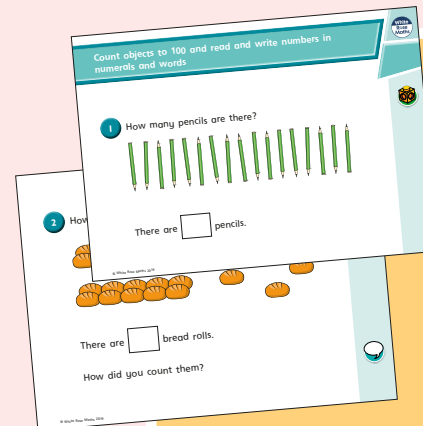
Worksheets to accompany every small step, providing relevant practice questions for each topic that will reinforce learning at every stage.



Display versions of the worksheet questions for front of class/whole class teaching.

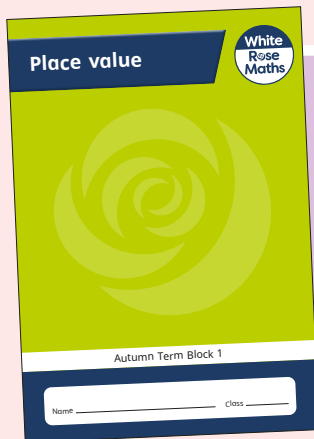


PowerPoint™ versions of the worksheet questions to incorporate them into lesson planning.



Answers to all the worksheet questions.

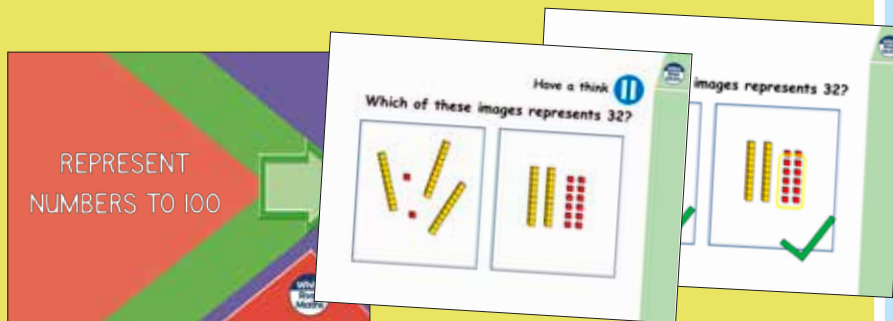
Question	Answer
1	There are 17 pencils.
2	There are 14 bread rolls. Children may have counted 3 tens and 1 roll.
3	twenty-eight
4	sixty-two
5	4 tens and 5 ones
6	a) seventeen b) twenty-one c) thirty-five d) eighty-two
7	a) 12 b) 80 c) 100 d) 9 e) 27 f) 14
8	79, 80, 81, 82, 83, 85 70, 79, 66, 64, 63
9	Eric has 20 sweets. Ed's friend gives her 7 sweets.



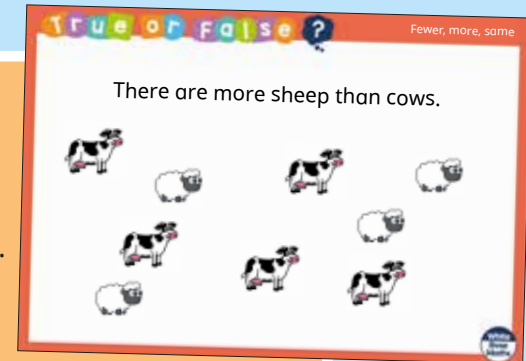
Also available as printed **workbooks**, per block.

Premium supporting materials

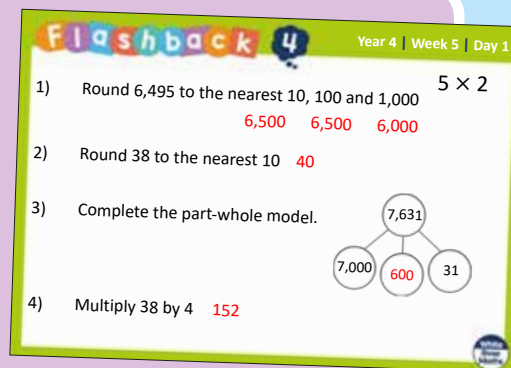
Teaching slides that mirror the content of our home learning videos for each step. These are fully animated and editable, so can be adapted to the needs of any class.



A **true or false** question for every small step in the scheme of learning. These can be used to support new learning or as another tool for revisiting knowledge at a later date.



Flashback 4 starter activities to improve retention. Q1 is from the last lesson; Q2 is from last week; Q3 is from 2 to 3 weeks ago; Q4 is from last term/year. There is also a bonus question on each one to recap topics such as telling the time, times-tables and Roman numerals.



Topic-based CPD videos

As part of our on-demand CPD package, our maths specialists provide helpful hints and guidance on teaching topics for every block in our schemes of learning.

Meet the characters

Our class of characters bring the schemes to life, and will be sure to engage learners of all ages and abilities. Follow the children and their class pet, Tiny the tortoise, as they explore new mathematical concepts and ideas.

White
Rose
Maths



Yearly overview

The yearly overview provides suggested timings for each block of learning, which can be adapted to suit different term dates or other requirements.

	Week 1	Week 2	Week 3	Week 4	Week 5	Week 6	Week 7	Week 8	Week 9	Week 10	Week 11	Week 12
Autumn	Number Place value		Number Addition, subtraction, multiplication and division				Number Fractions A		Number Fractions B		Measurement Converting units	
Spring	Ratio		Algebra		Number Decimals		Number Fractions, decimals and percentages		Measurement Area, perimeter and volume		Statistics	
Summer	Geometry Shape			Geometry Position and direction	Themed projects, consolidation and problem solving							

Autumn Block 1

Place value

Small steps

Step 1

Numbers to 1,000,000

Step 2

Numbers to 10,000,000

Step 3

Read and write numbers to 10,000,000

Step 4

Powers of 10

Step 5

Number line to 10,000,000

Step 6

Compare and order any integers

Step 7

Round any integer

Step 8

Negative numbers

Numbers to 1,000,000

Notes and guidance

In preparation for the next step (Numbers to 10,000,000), children recap their Year 5 learning by exploring numbers up to 1,000,000

Understanding that place value columns follow consistent patterns – ones, tens, hundreds, then (one) thousands, ten thousands, hundred thousands, before reaching millions – is key. Place value charts, Gattegno charts and place value counters can be used to support understanding of the relationships between columns and the construction of numbers.

Children also revise partitioning, exploring both standard and non-standard ways of composing numbers.

Writing numbers in words follows in Step 3

Things to look out for

- Children may find it difficult to conceptualise such large numbers, as they cannot easily be represented concretely and lie outside their experience.
- Children may think that place value columns go in the order ones, tens, hundreds, thousands, millions.
- Children may find numbers with several placeholders (for example, 500,020) difficult.

Key questions

- Where do the commas go when you write one million in figures?
- If 1,000,000 is the whole, what could the parts be?
- How else can you partition the number?
- What is the value of each digit in the number?
- Which columns will change if you add/subtract 10, 100, 1,000, ... to/from the number?
- When do you use placeholders in numbers?

Possible sentence stems

- The value of the _____ in _____ is _____
- The column before/after the _____ column is the _____ column.

National Curriculum links

- Read, write, order and compare numbers up to 10,000,000 and determine the value of each digit
- Solve number and practical problems that involve the above

Numbers to 1,000,000

Key learning

- What is the value of the digit 4 in each of the numbers in the place value chart?

Thousands			Ones		
H	T	O	H	T	O
		4	3	2	7
	3	5	4	0	2
2	4	7	1	9	8
8	1	2	5	4	3

- Complete the number sentences.
 - ▶ $604,821 = 600,000 + \underline{\hspace{1cm}} + \underline{\hspace{1cm}} + 20 + 1$
 - ▶ $\underline{\hspace{1cm}} = 300,000 + 4,000 + 700 + 4$
 - ▶ $2,000 + 8 + 60,000 + 500 + 700,000 = \underline{\hspace{1cm}}$
- Count up in 10,000s from 74,000 to 204,000
Count down in 100,000s from 1,000,000 to zero.
Count down in 100s from 9,312 to 7,812

- What number is shown in the place value chart?

Thousands			Ones		
H	T	O	H	T	O
● ●	● ● ● ● ● ● ● ● ● ●	●	● ● ● ● ● ● ● ● ● ● ● ● ● ●	● ● ●	● ● ● ● ● ● ● ● ●

What will the number be if you add four counters to the:

- tens column
 - ten-thousands column
 - hundreds column?
- Annie is using place value counters.
She has 4 ten-thousands counters, 12 thousands counters, 8 hundreds counters, 3 tens counters and 25 ones counters.
What is the greatest number she can make?
 - Fill in the missing numbers.
 $1 \text{ million} = 900,000 + \underline{\hspace{1cm}} = 990,000 + \underline{\hspace{1cm}} = \underline{\hspace{1cm}} + 999,000$

Numbers to 1,000,000

Reasoning and problem solving

100,000	200,000	300,000	400,000	500,000	600,000	700,000	800,000	900,000
10,000	20,000	30,000	40,000	50,000	60,000	70,000	80,000	90,000
1,000	2,000	3,000	4,000	5,000	6,000	7,000	8,000	9,000
100	200	300	400	500	600	700	800	900
10	20	30	40	50	60	70	80	90
1	2	3	4	5	6	7	8	9

What number is shown in the Gattegno chart?

Decrease the number shown by 30,000

Increase the number shown by 100,500

Challenge a partner to find other increases and decreases of the number.



463,528

433,528

564,028

Are the statements true or false?

Adding ten thousand to a number only ever changes the digits in exactly one column.

False

The number consisting of 70 thousands and 400 ones is 700,400

False

3 ten-thousands is the same as 30 thousands.

True

400 hundreds is the same as 4 ten-thousands.

True

A large number added to a large number is always a large number.

True

A large number subtracted from a large number is always a large number.

False

Numbers to 10,000,000

Notes and guidance

Children build on the previous step to explore numbers up to 10,000,000. They need to understand that the million can be considered a unit in the same way as the thousand. Numbers do not all have to be over 1,000,000 in this step; children should continue to experience smaller numbers alongside 7-digit numbers. The placement of commas and other separators should be discussed.

Familiar manipulatives and models, such as place value charts and counters, Gattegno charts and part-whole models, are used to represent numbers. Children partition the numbers in both standard and non-standard ways.

Things to look out for

- Children may struggle with where to position the commas in large numbers.
- Children may not recognise large numbers written with no commas.
- Unless they are confident with previous learning, children may think that place value columns go in the order ones, tens, hundreds, thousands, millions.
- Children may find numbers with several placeholders (for example, 1,006,020) difficult.

Key questions

- Where do the commas go when writing 7-digit numbers? How does this connect to place value charts?
- How does the place value chart help you to represent large numbers?
- What is the value of each digit in the number?
- Are 7-digit numbers always greater than 1,000,000?
- When do you use placeholders in numbers?
- What is the same and what is different about counting in 1,000s and counting in 1,000,000s?

Possible sentence stems

- The value of the _____ in _____ is _____
- The column before/after the _____ column is the _____ column.

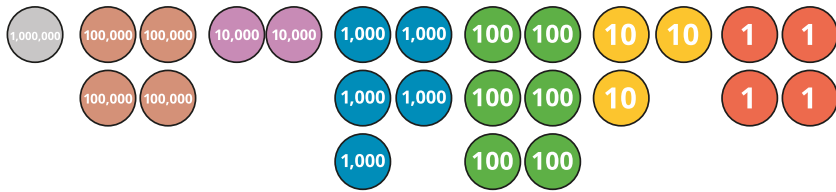
National Curriculum links

- Read, write, order and compare numbers up to 10,000,000 and determine the value of each digit
- Solve number and practical problems that involve the above

Numbers to 10,000,000

Key learning

- Count in 1,000,000s from zero to 10,000,000
- What number is represented?

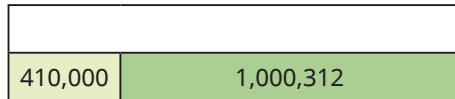


- Match the numbers to the representations.

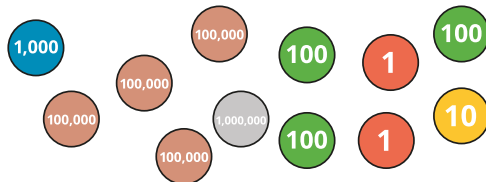
1,401,312

M	HTh	TTh	Th	H	T	O
●		●●●●	●	●●●	●	●●

1,041,312



1,410,312



- The meter shows the number of kilometres a car has travelled.



Ron writes the number as 3,678,42

Explain Ron's mistake.

- Here is a number in a place value chart.

Millions	Thousands			Ones		
O	H	T	O	H	T	O
4	2	8	7	2	9	5

What number is 300,000 greater than the number shown?

What number is 20,000 greater than the number shown?

- Count up in 10,000s from 463,500 to 1,000,500
- Count down in 10,000s from 463,500 to 3,500
- Count down in 1,000s from 463,500 to 433,500

Numbers to 10,000,000

Reasoning and problem solving

Jack has got some place value counters.

Some of my counters have a value of 1,000,000, some have a value of 10,000 and some have a value of 1



Jack picks four counters.

What different numbers greater than 1,000,000 could he make?

Jack wants to make a number greater than 5,000,000

What is the fewest number of counters he needs?

- 4,000,000
- 3,010,000
- 3,000,001
- 2,020,000
- 2,010,001
- 2,000,002
- 1,020,001
- 1,030,000
- 1,010,002
- 1,000,003

6 counters

Fill in the missing numbers.

$$824,309 = 800,000 + \underline{\hspace{2cm}} + 4,000 + 300 + 9$$

$$6,413,085 = \underline{\hspace{2cm}} + 80$$

$$58,904 = 50,000 + \underline{\hspace{2cm}} + 4$$

$$947,812 - 400,000 = \underline{\hspace{2cm}}$$

$$947,812 - 4,000 = \underline{\hspace{2cm}}$$

$$947,812 - 400 = \underline{\hspace{2cm}}$$

$$5,198,264 - \underline{\hspace{2cm}} = 5,098,264$$

$$5,198,264 - \underline{\hspace{2cm}} = 5,191,264$$

20,000

6,413,005

8,900

547,812

943,812

947,412

100,000

7,000

Read and write numbers to 10,000,000

Notes and guidance

Children should now be secure with the place value of numbers to 10,000,000. This small step develops their skill at reading and writing large numbers in words.

The focus of this step is learning the structure of how numbers are said and written in words, for example 4,378 as “four thousand, three hundred and seventy-eight” rather than just “four-three-seven-eight”. Using a comma as a separator helps children to read and write large numbers by tackling them in sections. This can be supported visually/concretely with place value charts, part-whole models or Gattegno charts.

Children should also be able to write numbers such as “half a million” in both words and numerals.

Things to look out for

- Children who find the “teen” numbers difficult may have problems with numbers such as 5,317,418
- Children may find reading and writing numbers with placeholders (for example, 5,208,001) difficult.

Key questions

- When a number is written with two commas, what does that tell you about the size of the number?
- What do the numbers before this comma represent?
- How do you write “one million” in words and numerals?
- How do you write “half a million” in words and numerals?
- When do we use “and” when reading or writing a number?

Possible sentence stems

- The digit before the first/second comma is _____
This part of the number is said/written as _____
- The digit after the first/second comma is _____
This part of the number is said/written as _____
- The whole of the number is said/written as _____

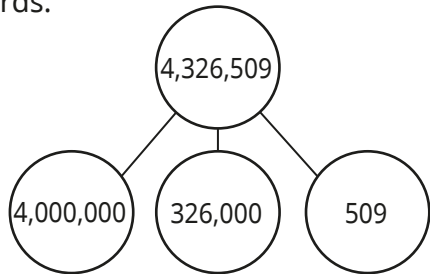
National Curriculum links

- Read, write, order and compare numbers up to 10,000,000 and determine the value of each digit
- Solve number and practical problems that involve the above

Read and write numbers to 10,000,000

Key learning

- Alex is using a part-whole model to help write the number 4,326,509 in words.

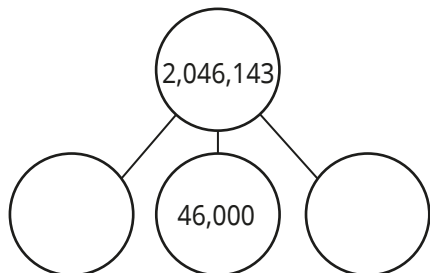


forty million and three hundred and twenty-six thousand and five hundred and nine

What mistakes has Alex made?

Write 4,326,509 correctly in words.

- Complete the part-whole model to show the number 2,046,143



Write the number 2,046,143 in words.

- Here is a number shown in a place value chart.

Millions	Thousands			Ones		
O	H	T	O	H	T	O
3	6	7	1	9	4	2

Write the number in words.

- A number is made up of 5 millions, 3 hundred-thousands, 7 tens and 9 ones.

Show the number on a place value chart.

Write the number in words and numerals.

- Write the numbers in numerals.

two million, eighty-three thousand and twelve

two million, eight hundred and three thousand and twenty

two million, eight hundred and twenty-three thousand and twelve

- Write 500,000 in words.
- Write the number "three and a half million" in numerals.

Read and write numbers to 10,000,000

Reasoning and problem solving

Use some of the digit cards and the clues to work out the number.



- The ten-thousands and hundreds columns have the same digit.
- The hundred-thousands digit is double the tens digit.
- The number has six digits.
- The number is less than six hundred and fifty-five thousand.

Find as many possible solutions, giving your answers in words and numerals.

Compare answers with a partner.



multiple possible answers, e.g.

650,533 – six hundred and fifty thousand, five hundred and thirty-three

Here is a number shown on a Gattegno chart.

1,000,000	2,000,000	3,000,000	4,000,000	5,000,000	6,000,000	7,000,000	8,000,000	9,000,000
100,000	200,000	300,000	400,000	500,000	600,000	700,000	800,000	900,000
10,000	20,000	30,000	40,000	50,000	60,000	70,000	80,000	90,000
1,000	2,000	3,000	4,000	5,000	6,000	7,000	8,000	9,000
100	200	300	400	500	600	700	800	900
10	20	30	40	50	60	70	80	90
1	2	3	4	5	6	7	8	9

Write in words the number that is:

- 80 greater than this number
- 80 less than this number
- 80,000 greater than this number
- 80,000 less than this number.

_____ six million, thirty thousand, five hundred and eighty-four

_____ six million, thirty thousand, four hundred and twenty-four

_____ six million, one hundred and ten thousand, five hundred and four

_____ five million, nine hundred and fifty thousand, five hundred and four

Powers of 10

Notes and guidance

Children should be confident with multiplying and dividing by 10, 100 and 1,000 from their learning in Year 5. In this small step, they use their place value knowledge to identify integers that are 10, 100, 1,000 times the size, or one-tenth, one-hundredth, one-thousandth the size of other integers. These relationships with decimal numbers are covered next term.

Children need to be aware that a value increases or decreases by a power of 10 between adjacent columns on a place value chart. They also need to realise that multiplying or dividing by 10 twice has the same effect as multiplying or dividing by 100 and that multiplying or dividing by 10 three times has the same effect as multiplying or dividing by 1,000

Place value charts and Gattegno charts are useful for modelling the effects of repeated multiplication and division by powers of 10

Things to look out for

- Children may think that the overall effect of, for example, $\times 10$ followed by $\times 10$ is $\times 20$
- The fact that numbers increase and decrease by a factor of 10 horizontally on a place value chart, but vertically on a Gattegno chart, may be confusing for children.

Key questions

- How can you tell if a number is a power of 10?
- Is this number a multiple of a power of 10? How can you tell?
- If you move a digit one/two places to the left in a place value chart, how many times greater is the value of the digit?
- How can you use a Gattegno chart to find a number 10 times/one-tenth the size of a given number?

Possible sentence stems

- _____ is 10 times the size of _____, so _____ is one-tenth the size of _____
- _____ is 100 times the size of _____, so _____ is one-hundredth the size of _____
- Multiplying/dividing by 10 twice/three times is the same as multiplying/dividing by _____

National Curriculum links

- Read, write, order and compare numbers up to 10,000,000 and determine the value of each digit
- Solve number and practical problems that involve the above

Powers of 10

Key learning

- What number is shown in the place value chart?

HTh	TTh	Th	H	T	O
		●● ●● ●	●● ●● ●● ●●	●●	●● ●● ●●

Multiply the number by 10 and show the answer in a place value chart.

What is the same and what is different?

Multiply the number by 100 and show the answer in a place value chart.

What is the same and what is different?

- Complete the statements.

_____ cm is the same length as 5,600 m.

_____ cm is the same length as 5,600 mm.

_____ m is the same length as 56,000 cm.

_____ m is the same length as 56,000 mm.

- What number is shown on the Gattegno chart?

1,000,000	2,000,000	3,000,000	4,000,000	5,000,000	6,000,000	7,000,000	8,000,000	9,000,000
100,000	200,000	300,000	400,000	500,000	600,000	700,000	800,000	900,000
10,000	20,000	30,000	40,000	50,000	60,000	70,000	80,000	90,000
1,000	2,000	3,000	4,000	5,000	6,000	7,000	8,000	9,000
100	200	300	400	500	600	700	800	900
10	20	30	40	50	60	70	80	90
1	2	3	4	5	6	7	8	9

Use the chart to make the number one hundred times the size of the number shown.

Use the chart to make the number one-hundredth the size of the number shown.

- Huan thinks that the number a thousand times the size of 2,500 is two and a half million.

Do you agree with Huan? Explain your answer.

- Which calculations have the same answers?

$$460 \times 10$$

$$46,000 \div 1,000$$

$$46 \times 10 \times 10$$

$$46 \times 100 \times 100$$

$$460 \times 10 \div 100$$

$$4,600 \div 10 \times 1,000$$

Powers of 10

Reasoning and problem solving

The Gattegno chart shows the answer to a calculation using powers of 10

1,000,000	2,000,000	3,000,000	4,000,000	5,000,000	6,000,000	7,000,000	8,000,000	9,000,000
100,000	200,000	300,000	400,000	500,000	600,000	700,000	800,000	900,000
10,000	20,000	30,000	40,000	50,000	60,000	70,000	80,000	90,000
1,000	2,000	3,000	4,000	5,000	6,000	7,000	8,000	9,000
100	200	300	400	500	600	700	800	900
10	20	30	40	50	60	70	80	90
1	2	3	4	5	6	7	8	9

Find two integer calculations using powers of 10 that give this answer.

Give your answers as calculations, for example:

_____ \times (or \div) _____ = _____ and sentences such as "_____ is 10 times (or one-tenth) the size of _____".

Compare answers with a partner.



various possible answers, e.g.

$$6,830 \times 10 = 68,300 \quad 68,300 \text{ is 10 times the size of } 6,830$$

$$6,830,000 \div 100 = 68,300$$

$$68,300 \text{ is one-hundredth the size of } 6,830,000$$

Annie is thinking of a number.



1,000 more
than my number
is 4,700



Annie

What number is 1,000 times the size of Annie's number?

3,700,000

Tommy is thinking of a number.



Tommy

The number
one-hundredth the
size of my number
is 38,746

What number is 100 less than Tommy's number?

3,874,500

Number line to 10,000,000

Notes and guidance

Children explore the number line to 10,000,000 using the unit of a million, making links to the familiar number lines to 10 and 10,000. They label partially filled number lines, identify points labelled on number lines and mark where a given number would lie on a number line.

Children should understand that half a million is equal to 500,000 and know that the midpoints between divisions on the number line to 10,000,000 can be written as, for example, “three and a half million” or “3,500,000”. This links to splitting different numbers and number lines into two, four, five and ten parts, which is also covered in this step.

Things to look out for

- Where number lines have more than one set of divisions, children may mix up the intervals between large divisions and smaller divisions.
- Children may confuse the number of intervals and the number of divisions.
- Children may not use the correct multiples when looking at midpoints, for example thinking the midpoint between 1,000,000 and 2,000,000 is 1,000,005

Key questions

- What are the values of the start and the end of the number line?
- What is each interval worth?
- How many small divisions are there between each of the large divisions on the number line? What is each small interval worth?
- What is the same and what is different about a number line that goes from 0 to 10,000 and a number line that goes from 0 to 10,000,000?
- What is the midpoint between _____ and _____?
- What is each interval worth if one million is split into two/four/five/ten equal parts?

Possible sentence stems

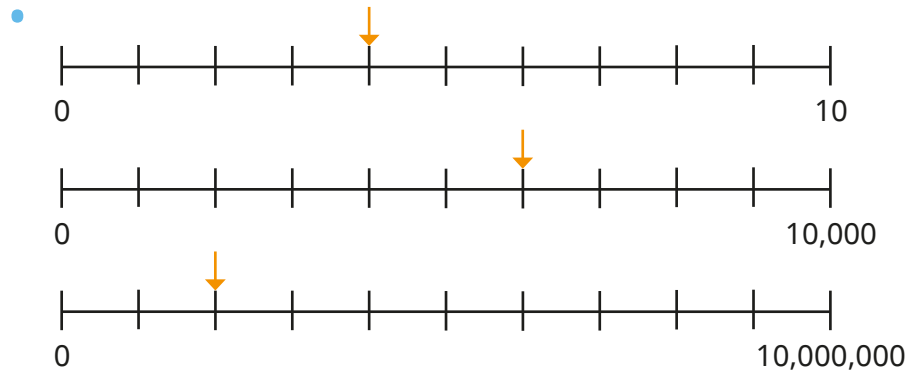
- The previous multiple of _____ is _____
- The next multiple of _____ is _____

National Curriculum links

- Read, write, order and compare numbers up to 10,000,000 and determine the value of each digit
- Solve number and practical problems that involve the above

Number line to 10,000,000

Key learning

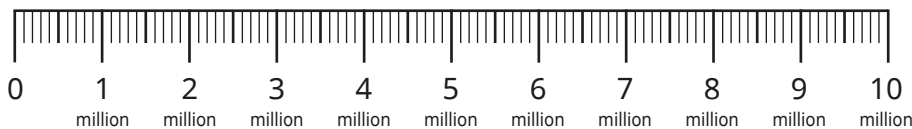


Label each division on the number lines.

What numbers are the arrows pointing to?

What is the same and what is different about the number lines?

- Here is a number line.

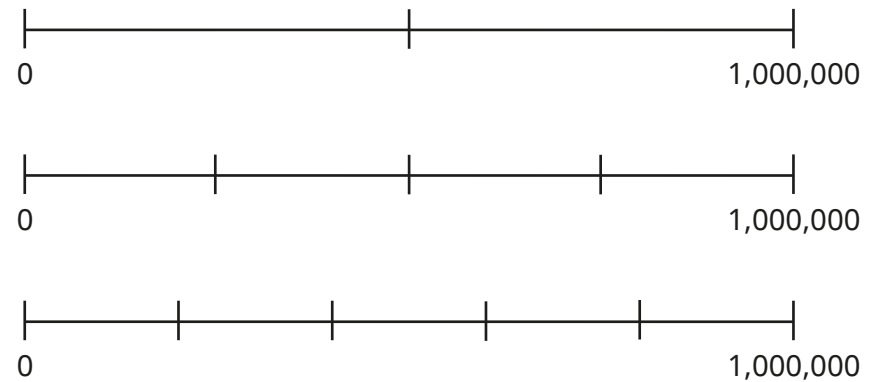


Draw arrows to show the positions of these numbers on the number line.

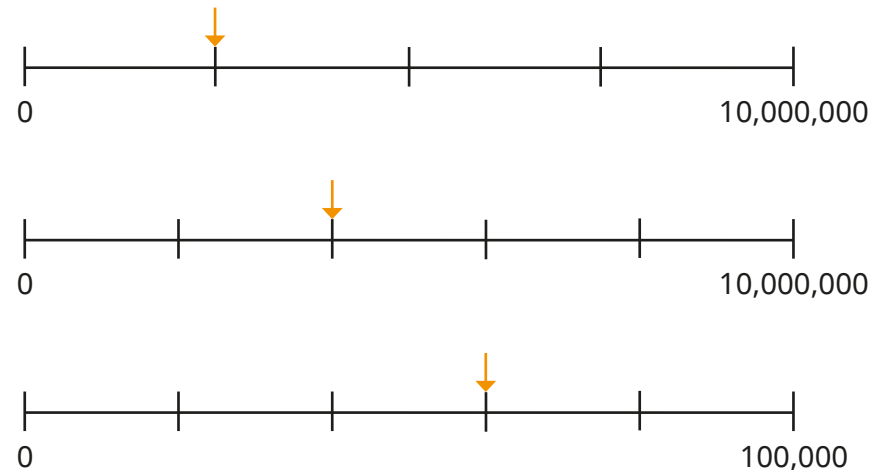
- | | | | |
|-----------|-------------------------|-----------|-----------|
| 1,500,000 | five and a half million | 6,200,000 | 8,950,000 |
|-----------|-------------------------|-----------|-----------|

Which numbers can you place more accurately than others?

- Label the divisions on each number line.



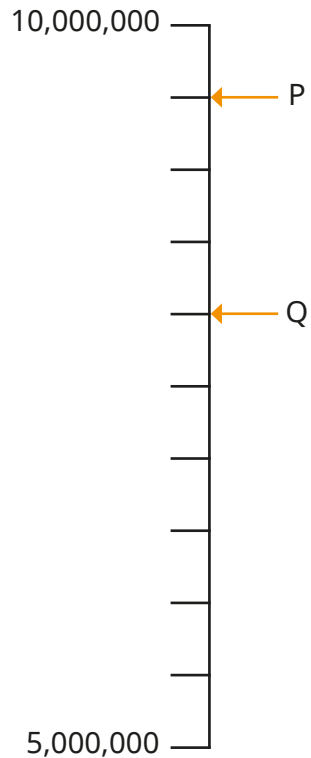
- What numbers are the arrows pointing to?



Number line to 10,000,000

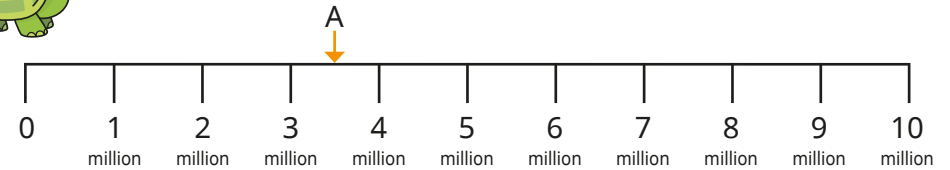
Reasoning and problem solving

Find the difference between P and Q.



1,500,000

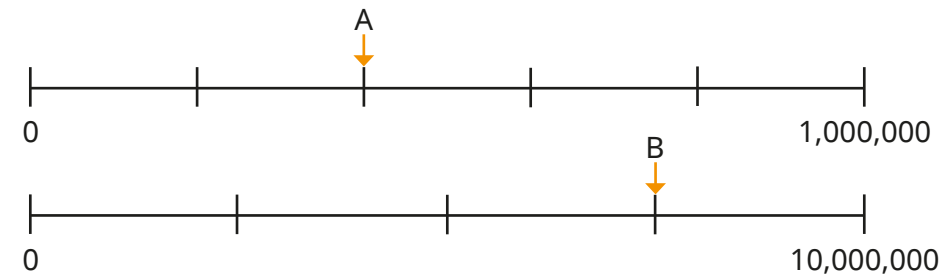
Compare methods with a partner.



Tiny says A is pointing to 3,050,000

Explain the mistake that Tiny has made.

Tiny has incorrectly found the midpoint of 3 and 4 million.



Work out $B - A$.

7,100,000

Compare and order any integers

Notes and guidance

In Year 5, children learned how to compare and order integers up to 1,000,000. This small step extends their learning to integers up to 10,000,000

Children compare numbers with the same number of digits, and with different numbers of digits, using their knowledge of place value columns. They present numbers in a variety of forms and use these different representations to aid their understanding when comparing and ordering.

Encourage the use of inequality symbols and precise mathematical language such as “greater than” and “less than”.

Things to look out for

- Children may just look at the size of the leading digits and not consider the place value of the digits within the numbers.
- Children may need to be reminded of the meanings of the words “ascending” and “descending”.
- Children may need to be reminded about inequality symbols and their meanings.

Key questions

- What is the value of each digit in the number?
- Which digit in each number has the greatest value? What is the value of these digits?
- When comparing two numbers with the same number of digits, what do you look at first?
- What is the difference between ascending and descending order?
- What is different about comparing numbers with the same number of digits and comparing numbers with different numbers of digits?

Possible sentence stems

- The value of the first digit in the number _____ is _____
- _____ is less than/greater than _____

National Curriculum links

- Read, write, order and compare numbers up to 10,000,000 and determine the value of each digit
- Solve number and practical problems that involve the above

Compare and order any integers

Key learning

- Which is the greater number in each pair?

▶	62,800	60,820
▶	247,612	247,162
▶	8,642,371	8,643,271

Explain how you know.

- Complete the statements to make them true.

M	HTh	TTh	Th	H	T	O	○	M	HTh	TTh	Th	H	T	O	
●●	●●	●●	●	●●	●	●●		●●	●	●●	●●	●●	●	●●	
M	HTh	TTh	Th	H	T	O	>	M	HTh	TTh	Th	H	T	O	
●		●●●	●	●●	●●	●●									

- Write the numbers in ascending order.

6,503,102 651,300 6,550,021 690,210

- Which calculation has the greater answer?

$600,000 + 50,000 + 7,000$	$400,000 + 256,000$
----------------------------	---------------------

- Write $<$, $>$ or $=$ to make the statements correct.

62,520 ○ 602,250

3,218,000 ○ 399,875

426,000 ○ forty-four thousand

990,099 ○ one million

- Here are three numbers ordered from the greatest to the smallest, but one number has been covered up.

three hundred and thirteen thousand and thirty-three

✖

○

250,000

53,033

What might the covered number be?

Compare and order any integers

Reasoning and problem solving

Eva has put eight 6-digit numbers in ascending order.



- The first number in her list is 345,900
- The last number in her list is 347,000
- All the other numbers in her list have a digit sum of 20
- None of the numbers in her list have any repeated digits.

Find the other six numbers in Eva's list and write them in ascending order.

346,025
346,052
346,205
346,250
346,502
346,520

$$\underline{\hspace{2cm}} + 80,000 < \text{half a million}$$

Complete the sentences.

The missing number could be _____

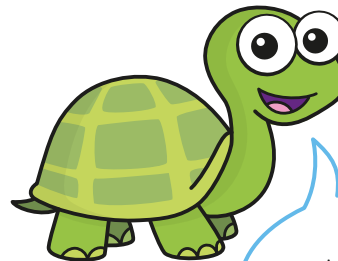
The missing number cannot be _____

The missing number must be _____

any number less than 420,000, e.g. 10,000

any number greater than or equal to 420,000, e.g. 600,000

multiple possible answers, e.g. less than 420,000



56,700 is greater than 201,000 because 5 is greater than 2

Explain the mistake that Tiny has made.

Tiny hasn't considered the place value of the digits.

Round any integer

Notes and guidance

In Year 5, children learned to round any number up to 1,000,000 to any power of 10 up to 100,000. This small step reviews and builds on this concept so that children also learn to round to the nearest million.

Children need to be confident with identifying the previous and next multiples of the appropriate power of 10 of the number, and finding the midpoints of those multiples. Number lines are useful as support here, as children can identify which multiple the number is closer to.

Children may need reminding that when a number is exactly halfway between two successive multiples the convention is to round to the greater multiple.

Things to look out for

- Children may be confused by the language “round down”/“round up” and round 428,513 to 328,513 (or 300,000) to the nearest 100,000
- Children may look at the digit of the rounding rather than the next digit, for example, looking at the thousands column rather than the hundreds when rounding to the nearest thousand.

Key questions

- Which multiples of 1,000,000 does the number lie between?
- How can you represent the rounding of this number on a number line?
- Which division on the number line is the number closer to?
- What is the number rounded to the nearest million?
- What is the most appropriate way of rounding this number?
- Which place value column should you look at to round the number to the nearest ten/hundred/thousand/ten thousand/hundred thousand/million?

Possible sentence stems

- The previous multiple of _____ is _____
- The next multiple of _____ is _____
- _____ rounded to the nearest _____ is _____

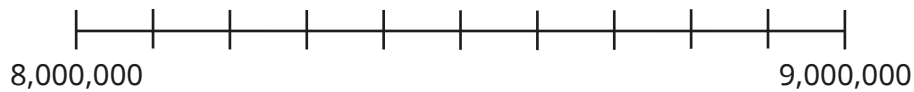
National Curriculum links

- Round any whole number to a required degree of accuracy
- Solve number and practical problems that involve the above

Round any integer

Key learning

-



Draw an arrow to show the approximate position of 8,640,000 on the number line.

Round 8,640,000 to the nearest million.

- The population of London is 8,982,604
Between which two multiples of 1,000,000 does this number lie?
Round the population of London to the nearest million.

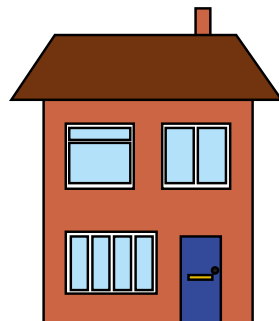
- In April 2021, the average price of a house in England was £273,486

Round this price to the nearest £100,000

Round this price to the nearest £10,000

Round this price to the nearest £1,000

Which do you think is the most appropriate number to round the price to?



-

HTh	TTh	Th	H	T	O
●● ●● ●		●● ●● ●● ●	●● ●● ●● ●● ●	●● ●● ●●	●● ●● ●● ●● ●

Round the number in the place value chart to:

- the nearest ten thousand
- the nearest hundred thousand
- the nearest million.

-



My number rounds to 38,000 to the nearest thousand.


What is the greatest possible value of Dexter's number?

What is the smallest possible value of Dexter's number?

Round any integer

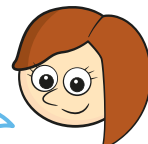
Reasoning and problem solving

Mo and Rosie are each thinking of a number.



My number is 1,350,000 when rounded to the nearest ten-thousand.

Mo



My number is 1,000,000 when rounded to the nearest million.

Rosie

Both numbers are whole numbers.

What is the greatest possible difference between the two numbers?

854,999
(if Mo's number is 1,354,999 and Rosie's number is 500,000)

Four children each have one of these cards.

15,987	15,813
15,101	16,101

Each child gives a clue about the number on their card.

Filip says, "My number rounds to 16,000 to the nearest thousand."

Esther says, "My number has 1 hundred."

Jack says, "My number is 15,990 when rounded to the nearest ten."

Dora says, "My number is 15,000 when rounded to the nearest thousand."

Match the cards to the children.

Filip: 15,813
Esther: 16,101
Jack: 15,987
Dora: 15,101

Negative numbers

Notes and guidance

Children encountered negative numbers in Year 5. The focus of this small step is using negative numbers in real-life contexts while reinforcing children's understanding of the number line extending beyond zero.

Both horizontal and vertical number lines should be used, with the vertical line linking to reading temperatures on a thermometer. As well as adding and subtracting from positive and negative numbers, children learn to find the difference between numbers, including calculating intervals across zero. At this stage, children do not need to subtract negative numbers, so there is no need to cover calculations of the form $7 - -5$.

A recap of the Year 5 steps relating to this topic may be useful.

Things to look out for

- When calculating intervals, children may count the divisions rather than the number of intervals.
- Children may have heard “rules” such as “two minuses make a plus” and mistakenly think that, for example, $-3 - 2 = +5$
- Because 5 is greater than 3, children may think that -5 is greater than -3

Key questions

- What is the same and what is different about the numbers 2 and -2 (negative two)?
- How far is -5 from zero? How far is -5 from 1?
- Which is the greater temperature, -1 degrees or -2 degrees?
- How do you find the difference between two negative numbers?
- How do you find the difference between a positive number and a negative number?
- What is the same and what is different about counting forwards/backwards along a number line beyond zero?

Possible sentence stems

- To find the number _____ greater/less than _____, I count _____ on the number line.
- _____ is _____ away from zero.

National Curriculum links

- Use negative numbers in context, and calculate intervals across zero
- Solve number and practical problems that involve the above

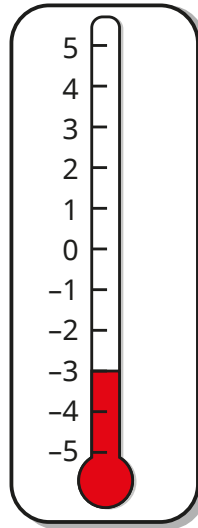
Negative numbers

Key learning

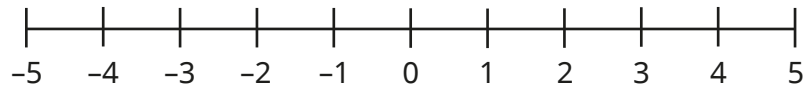
- What temperature does the thermometer show?

If the temperature drops by 1°C , what temperature will the thermometer show?

What temperature is 5°C warmer than the temperature shown on the thermometer?



- Use the number line to answer the questions.



What is 6 less than 4?

What is 5 more than -2 ?

What is the difference between 3 and -3 ?

- The table shows the temperatures in four places on a day in January.

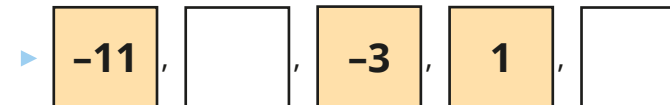
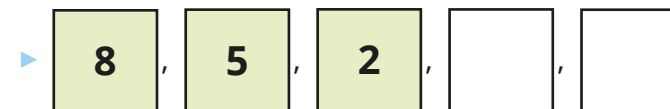
Bradford	2°C
Harlow	-3°C
Aberdeen	-7°C
Southampton	4°C

Which place has the lowest temperature?

Work out the difference between the temperature in Harlow and the temperature in Southampton.

The next day the temperature in Bradford dropped by 6°C . Work out the new temperature in Bradford.

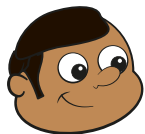
- Complete the number sequences.



Negative numbers

Reasoning and problem solving

A company has plans to construct a building with floors above and below ground.



If we build from floor -10 to floor 10, we will have 20 floors in total.

Do you agree? Explain your answer.

No
There will be 21 floors as you need to include floor zero.

Find different ways of completing the calculation.

$$\underline{\quad} + \underline{\quad} = -2$$

multiple possible answers, e.g.
 $-6 + 4$ $-80 + 78$
 $-5 + 3$ $-2 + 0$

Is each statement always true, sometimes true or never true?

When you count forwards in tens from a positive 1-digit number, the final digits of all the numbers are the same.

When you count backwards in tens from a positive 1-digit number, the final digits of all the numbers are the same.

Give examples to support your answers.

What patterns can you see?

The first statement is always true (e.g. 8, 18, 28, 38 ...). Adding tens does not affect the ones column.

The second statement is sometimes true. It is true when we start at 5 (5, -5, -15, -25 ...), but false from every other number (e.g. 8, -2, -12, -22 ... or 7, -3, -13, -23 ...).



Autumn Block 2

**Addition, subtraction,
multiplication and division**

Small steps

Step 1

Add and subtract integers

Step 2

Common factors

Step 3

Common multiples

Step 4

Rules of divisibility

Step 5

Primes to 100

Step 6

Square and cube numbers

Step 7

Multiply up to a 4-digit number by a 2-digit number

Step 8

Solve problems with multiplication

Small steps

Step 9

Short division

Step 10

Division using factors

Step 11

Introduction to long division

Step 12

Long division with remainders

Step 13

Solve problems with division

Step 14

Solve multi-step problems

Step 15

Order of operations

Step 16

Mental calculations and estimation

Small steps

Step 17

Reason from known facts

Add and subtract integers

Notes and guidance

This small step reviews and extends children's learning of how to add and subtract integers with any number of digits.

Children use the formal column method for numbers with the same and different numbers of digits. They also practise mental strategies with both large and small numbers, using their understanding of place value.

Children solve multi-step problems, choosing which operations and methods to use based on the context of the problem and the types of numbers involved.

The use of concrete manipulatives can support children's understanding, especially where exchanges are required.

Things to look out for

- Children may not line the numbers up correctly when setting out an addition or a subtraction.
- Children may try to use formal methods when mental strategies would be more appropriate, for example adding 999 is more easily done by adding 1,000 and then subtracting 1
- When solving multi-step problems, children may need support to choose the type and order of operations needed.

Key questions

- What is the greatest digit you can have in a place value column?
- How do you exchange when adding?
- How do you exchange when subtracting?
- Which columns are affected by the exchange?
- How do you know whether to add or subtract the numbers?
- How can you check your answer to the calculation?

Possible sentence stems

- In column addition/subtraction, we start with the _____ place value column.
- The _____ is in the _____ column. It represents _____

National Curriculum links

- Solve addition and subtraction multi-step problems in contexts, deciding which operations and methods to use and why
- Solve problems involving addition, subtraction, multiplication and division
- Use estimation to check answers to calculations and determine, in the context of a problem, an appropriate degree of accuracy

Add and subtract integers

Key learning

- Work out the additions.

		6	2	3	
	+	3	5	8	
<hr/>					

		5	6	4	7	
	+		8	6	1	
<hr/>						

		3	4	6	0	8	
	+	2	9	0	8	7	
<hr/>							

- Work out the subtractions.

		7	5	2	
	-	3	1	5	
<hr/>					

		8	1	6	
	-	5	3	9	
<hr/>					

		3	4	6	0	8	
	-	1	2	7	2	7	
<hr/>							

- Find the answers to the calculations.

		3	4	6	2	1	
	+	2	5	7	3	4	
<hr/>							

		4	7	6	1	3	2	5	
	-		9	3	8	0	5	2	
<hr/>									

- Which calculations would you work out mentally, and which would you work out using the column method?

$67,832 + 5,258$

$834,501 - 299,999$

$450,000 + 201,000$

8 million subtract $3\frac{1}{2}$ million

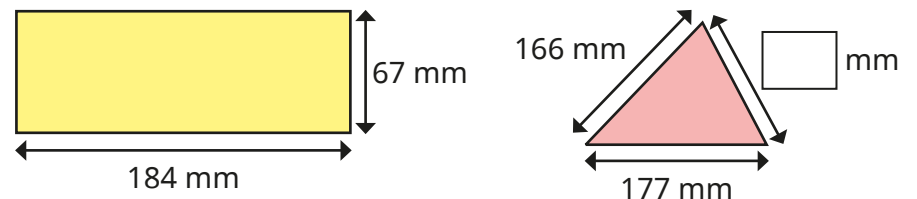
$604,000 - 25,000$

Work out the answers to the calculations.

- Find the missing digits.

		5	2	2	4	7	
	+	3		5	9	0	4
<hr/>							
		9	0		3		2

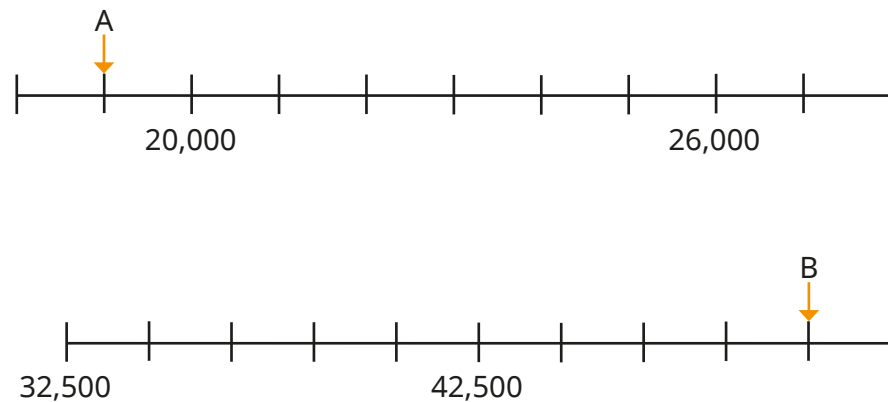
- The perimeter of the triangle is equal to the perimeter of the rectangle. Work out the unknown length of the triangle.



Add and subtract integers

Reasoning and problem solving

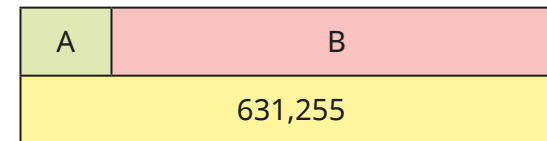
Find the difference between A and B.



Explain your method to a partner.

31,500

Here is a bar model.



- A is an odd integer that rounds to 100,000 to the nearest 10,000
- The sum of the digits of A is 30
- B is an even integer that rounds to 500,000 to the nearest 100,000
- The sum of the digits of B is 10
- A and B are both multiples of 5

What could be the values of A and B?

Explain your reasoning to a partner.

multiple possible answers, e.g.

A = 99,255

B = 532,000

Common factors

Notes and guidance

This small step reinforces children's understanding of factors and common factors, introduced in Years 4 and 5 respectively.

Some children may still choose to use arrays and other representations, but knowledge of times-tables and the use of familiar rules of divisibility are to be encouraged. The rules of divisibility will be reviewed again later in the block.

Children work systematically to find the complete list of factors of a number, and learn to use their knowledge that factors usually come in pairs to spot missing factors.

Children are not required to formally identify the highest common factor of two or more numbers, but can be extended to consider this idea.

Things to look out for

- Children may confuse the ideas of factors and multiples.
- Children may not be familiar with the use of the word "common" in this context.
- Errors may be made with times-tables, resulting in incorrect factors.
- Children may forget 1 and the number itself when listing factors.

Key questions

- What are the factors of _____?
- What factors do _____ and _____ have in common?
- How can you easily tell if 2/5/10 is a factor of a number?
- If you know one factor of a number, how can you use it to find another factor of the number?
- Is 1 a factor of all numbers?
- How can you work systematically to find all the factors of a number?

Possible sentence stems

- _____ is a factor of all numbers.
- The largest factor of a number is always _____
- _____ is a factor of _____ because _____ is in the _____ times-table.

National Curriculum links

- Identify common factors, common multiples and prime numbers
- Solve problems involving addition, subtraction, multiplication and division

Common factors

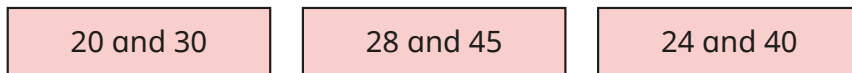
Key learning

- List the factors of 24

List the factors of 36

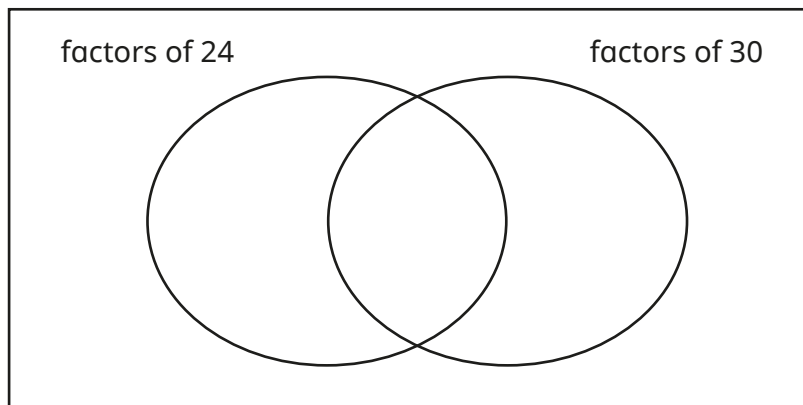
What are the common factors of 24 and 36?

- Find the common factors of each pair of numbers.



- Write the numbers in the sorting diagram.

1 2 3 4 5 6 8 10 12 15 24 30



List the common factors of 24 and 30

- Decide if each statement is true or false.

5 is a factor of both 95 and 75

3 is a common factor of 45 and 54

4 is not a common factor of 56 and 80

- Here is a table for sorting numbers.

Write one number in each box.

	Factor of 6	Not a factor of 6
Factor of 9		
Not a factor of 9		

Compare answers with a partner.

- Find the common factors of 300, 400 and 500
- The common factors of two numbers are 1, 3 and 5
What could the two numbers be?

Common factors

Reasoning and problem solving

A fruit stall has 49 pears and 56 oranges.



The pieces of fruit are put into boxes with an equal number of pears or oranges in each box.

Tiny

There will be 8 pieces of fruit in each box.



There will be 7 pieces of fruit in each box.



Jack

Who is correct, Tiny or Jack?

Explain how you know.



Jack

Brett has two pieces of string.



One is 160 cm long and the other is 200 cm long.

He cuts them both into smaller pieces.

All the pieces are the same length.

What are the possible lengths of the smaller pieces of string?

1 cm, 2 cm, 4 cm,
5 cm, 8 cm, 10 cm,
20 cm, 40 cm

Dani has 54 red sweets and 45 green sweets.



She puts them into bags so that each bag has an equal number of red sweets and an equal number of green sweets.

What is the greatest number of bags she can make?

How many sweets of each colour will there be in each bag?

9 bags, each with
6 red sweets and
5 green sweets

Common multiples

Notes and guidance

Children are familiar with the idea of multiples of numbers from earlier study of times-tables. Building on this knowledge, they now find common multiples of two or more numbers.

As with factors, arrays and other representations may still be used as support, but knowledge of times-tables is key. Some multiples can be recognised using the rules of divisibility, which are explored in detail in the next small step.

Encourage children to work systematically to find lists of multiples rather than just finding the product of the given numbers, as this may miss some multiples.

Children do not need to be able to formally identify the lowest common multiple of two or more numbers, but can be challenged to consider the first common multiple of a pair of numbers.

Things to look out for

- Children may confuse the ideas of factors and multiples.
- Errors may be made with times-tables, resulting in incorrect factors.
- A common misconception is that the only common multiple of a pair of numbers is the product of the numbers.

Key questions

- How do you find the multiples of a number?
- What multiples do _____ and _____ have in common?
- What is the difference between a multiple and a factor?
- Can a number be both a factor and a multiple of another number?
- How can you tell if a number is a multiple of 2/5/10?
- When do numbers have common multiples that are less than their product?

Possible sentence stems

- The first multiple of a number is always _____
- _____ is a multiple of _____ because _____ \times _____ = _____
- _____ is a common multiple of _____ and _____

National Curriculum links

- Identify common factors, common multiples and prime numbers
- Solve problems involving addition, subtraction, multiplication and division

Common multiples

Key learning

- Here is a hundred square.

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

Shade the multiples of 6

Circle the multiples of 5

What common multiples of 5 and 6 do you find?

Use these numbers to find other common multiples of 5 and 6

- Find the first three common multiples of each pair of numbers.

4 and 5

5 and 6

4 and 8

6 and 8

- Find five common multiples of 4 and 3

- Here is a table for sorting numbers.

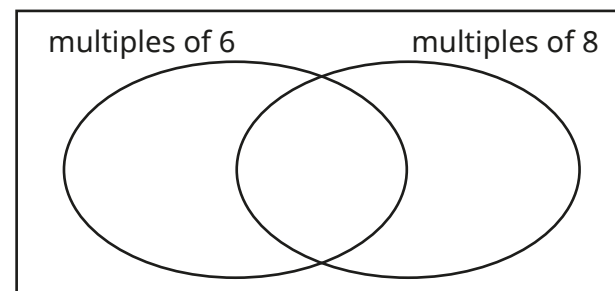
Write one number in each box.

	Multiple of 8	Not a multiple of 8
Multiple of 5		
Not a multiple of 5		

Compare answers with a partner.

- Write the numbers in the sorting diagram.

12 18 40 6 48 24 16 42 56 54 30



- Nijah plays football every 4 days and Kim plays football every 6 days.

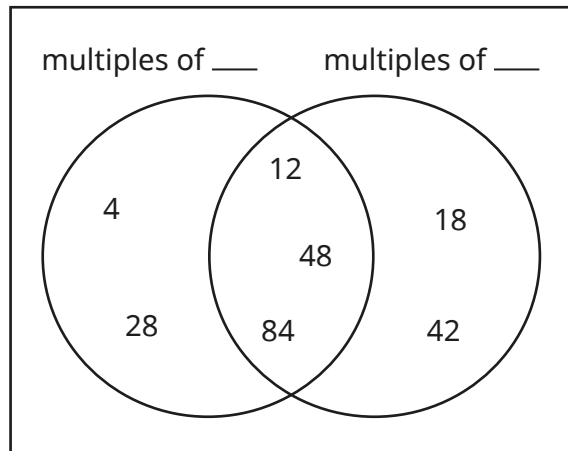
They both played football today.

In how many days will they next both play football on the same day?

Common multiples

Reasoning and problem solving

Complete the labels of the sorting diagram.



Write another number in each section.
 Find a square number that will go in the middle section.
 Compare answers with a partner.



various possible answers, e.g. multiples of 4, multiples of 6

multiple possible answers, e.g. 40, 72, 66

36, 144

Ms Fisher's age is double her sister's age.



They are both older than 20 but younger than 50

Their ages are both multiples of 7

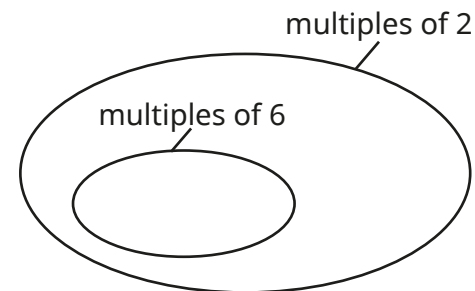
What are their ages?

Ms Fisher is 42 and her sister is 21

Write the numbers in the sorting diagram.



- 10
- 12
- 14
- 16
- 18
- 20



multiples of 2:
10, 12, 14, 16, 18, 20
 multiples of 6:
12, 18

Rules of divisibility

Notes and guidance

Children should be familiar with most rules of divisibility from looking at patterns in times-tables in their earlier learning and the previous two steps.

Children recognise divisibility by 2, 5 or 10 by looking at the ones digits of a number. They know a number is divisible by 4 if halving the number gives an even result and the corresponding rule for divisibility by 8. They know that numbers are divisible by 3 if the sum of their digits is divisible by 3, and divisible by 9 if the sum of their digits is divisible by 9

Children now learn to combine these rules to deal with other potential factors, for example to be divisible by 6 a number must be divisible by both 2 and 3

Children should recognise that a 2-digit number is divisible by 11 if the digits are the same.

Things to look out for

- Children may over-generalise rules, for example incorrectly applying the digit-sum rule for 3 and 9 or the final-digit rule for 5 to other numbers.
- Children may need support in understanding the combining of rules such as “a number is divisible by 12 if it is divisible by both 3 and 4”

Key questions

- How does the ones digit help you to decide if a number is divisible by 2, 5 or 10?
- How can you use the rule for divisibility by 2 to find out if a number is divisible by 4/8?
- What two other numbers must a number be divisible by if the number is divisible by 6/12?
- How can you tell if a 2-digit number is divisible by 11?
- Which divisibility rules are based on the sum of the digits of a number?

Possible sentence stems

- If a number is divisible by _____ and _____, then the number must also be divisible by _____
- If the sum of the digits is divisible by _____, then the number is divisible by _____
- A number is divisible by _____ if its ones digit is _____

National Curriculum links

- Solve problems involving addition, subtraction, multiplication and division

Rules of divisibility

Key learning

- Which of the numbers are divisible by 2?

62	901	5,462
10,308	111,111	224,528

Which of the numbers are also divisible by 4? How can you tell?

- Use the digit sums to decide which numbers are divisible by 3 and which are also divisible by 9

78	801	5,460
12,307	555,222	48,117

- Find a number that matches each description.

a 3-digit number that is divisible by 5

a 6-digit number that is divisible by 10

a 4-digit number that is divisible by 5 and 3

a 5-digit number that is divisible by 3 but not divisible by 5

- Scott is packing cakes into boxes.

He puts an equal number of cakes into each box with no cakes left over.

He has 1,032 cakes to pack.

How many cakes can go in each box?



- Use ticks and crosses to complete the table.

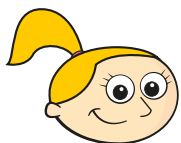
	Is the number divisible by ...?				
	3	4	6	9	11
87					
96					
99					
216					
702					

- The children at a school all have lunch at the same time. There are 672 children and an equal number of them sit at each table. No more than 12 children sit at a table. How many tables could there be?

Rules of divisibility

Reasoning and problem solving

The year number of a leap year is divisible by 4



If the final two digits of a number are divisible by 4, then the number itself is divisible by 4

Use Eva's rule to find out which of these years were, or will be, leap years.

1536

1674

1928

1992

2024

2050

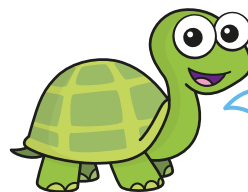
2062

2956

Why does this rule work?

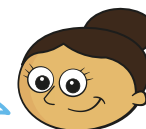
1536, 1928, 1992, 2024, 2956

Tiny and Dora are talking about rules for division.



Tiny

If a number is divisible by 10, then it must also be divisible by 5



Dora

If a number is divisible by 5, then it must also be divisible by 10

Tiny is correct.

Dora is incorrect.

Do you agree with Tiny and Dora?

Explain your answer.

Primes to 100

Notes and guidance

Children first encountered prime numbers and composite numbers in Year 5. This small step reviews that learning and develops children's knowledge of factors so that they can deepen their understanding of prime numbers.

Children recognise that a number is prime when it has exactly two factors: 1 and itself. They also look at identifying the prime factors of a given number.

By the end of this step, children should be able to identify all the primes less than 100 and recall at least the primes to 19

Children should be familiar with square and cube numbers from earlier years, so this is something that can be revisited here, but is also covered in detail in the next small step.

Things to look out for

- A common misconception is that 1 is a prime number.
- Children may think that all prime numbers are odd and not realise that 2 is a prime number.
- Numbers that are outside times-tables knowledge (e.g. 51) may be mistakenly thought of as prime. Encourage children to use divisibility rules from the previous step to check these.

Key questions

- What is a prime number?
- What is a composite number?
- How many factors does a prime number have?
- Why is 1 not a prime number?
- How can you find the prime factors of a number?
- Are the multiples of prime numbers also prime?

Possible sentence stems

- The factors of _____ are _____
The prime factors of _____ are _____
- _____ is prime because it has exactly _____ factors.
- _____ is a composite number because _____ = _____ × _____

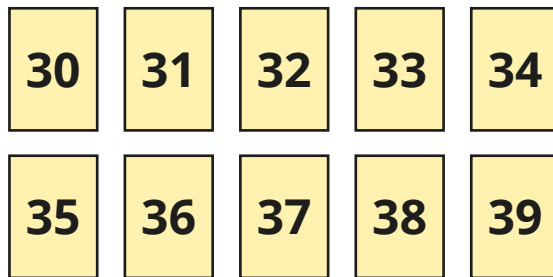
National Curriculum links

- Identify common factors, common multiples and prime numbers
- Solve problems involving addition, subtraction, multiplication and division

Primes to 100

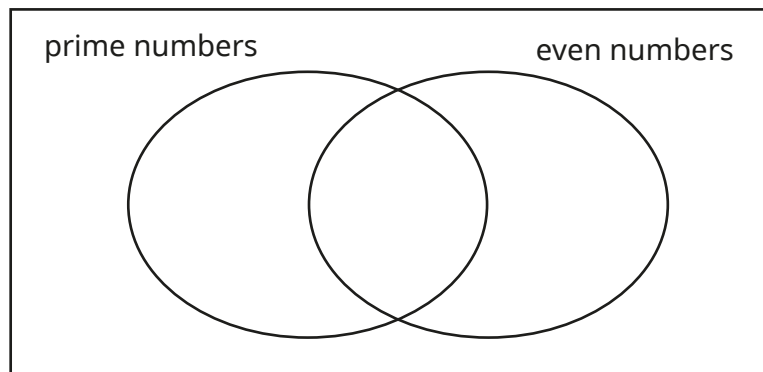
Key learning

- List all the prime numbers that are less than 20
- Which of these numbers are prime and which are composite?



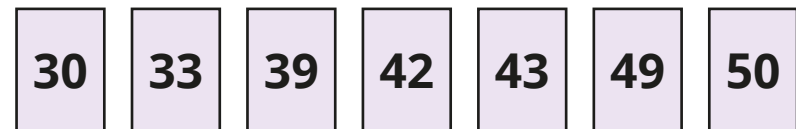
- Explain how you know 51 is a composite number.
- Write the numbers in the sorting diagram.

10 13 2 12 11 6 7



- List the factors of 20
Which factors of 20 are prime?


- Find the prime factors of the numbers.




- The sum of two prime numbers is 36
What might the numbers be?
How many different answers can you find?
- Write the three prime numbers that multiply to make 105
_____ × _____ × _____ = 105
- List the numbers from 40 to 49
Which of the numbers are prime?
Which of the numbers are square?
Which of the numbers are composite?

Primes to 100

Reasoning and problem solving


Ron is thinking of a number. 

 I am thinking of a number greater than 10


Use the clues to work out Ron's number.


- It is a composite number.
- It has two prime factors.
- It is an odd number.
- It is a factor of 60

15

Shade the multiples of 6 on a hundred square. 

What do you notice about all the numbers either side of the multiples of 6?

 I think that there is always a prime number next to a multiple of 6

Is Whitney correct?
Explain your reasoning. 

All the numbers next to a multiple of 6 are odd.

Yes

Square and cube numbers

Notes and guidance

Children encountered square and cube numbers in Year 5, and this small step revisits that learning and the notation for squared (2) and cubed (3).

The concept of square and cube numbers can be supported by making links to area and volume (the formula for the volume of a cuboid will be covered next term).

Children explore the factors of square and cube numbers, noticing that square numbers always have an odd number of factors, but cube numbers can have an odd or even number of factors.

The vocabulary of earlier small steps in this block, such as “factor”, “multiple” and “prime” can also be reinforced at this stage.

Things to look out for

- Children may confuse the idea of squaring/cubing with multiplying by $2/3$
- Children may not realise that 1 is both a square number and a cube number.

Key questions

- How do you square a number?
- How do you cube a number?
- Are the squares of even/odd numbers even or odd?
- Are the cubes of even/odd numbers even or odd?
- Can a number be both a square number and a cube number?
- How can you use a square number to help find a cube number?

Possible sentence stems

- To square a number, you multiply the number by _____
- To cube a number, you multiply the number by _____ and then by _____ again.
- I know _____ is a square/cube number because ...

National Curriculum links

- Solve problems involving addition, subtraction, multiplication and division

Square and cube numbers

Key learning

- The table shows some square numbers and cube numbers.

Complete the table and describe any patterns and connections you notice. The first row has been done for you.

1^2	1×1	1	1^3	$1 \times 1 \times 1$	1
					8
	3×3		3^3		27
	4×4			$4 \times 4 \times 4$	
		25	5^3		
				$6 \times 6 \times 6$	
8^2					

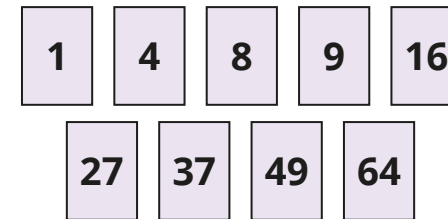
- Write $>$, $<$ or $=$ to make the statements correct.

$$3^3 \bigcirc 4^2$$

$$8^2 \bigcirc 4^3$$

$$11^2 \bigcirc 5^3$$

- Here are some number cards.



Which numbers are square?

Which numbers are cube?

Which numbers are both square and cube?

Which numbers are prime?

- List the factors of the first five square numbers.

How many factors do they each have?

What do you notice about the number of factors a square number has?

Is the same true for cube numbers?

- $\bullet + \blacktriangle = 38$

\bullet is a cube number.

\blacktriangle is a prime number.

Find pairs of values for \bullet and \blacktriangle .

Square and cube numbers

Reasoning and problem solving

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

Shade all the square numbers.

Use a different colour to shade the multiples of 4

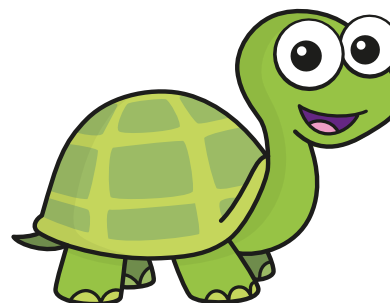
What do you notice?



Square numbers are always a multiple of 4 or one greater than a multiple of 4



Square numbers only end in 1, 4, 5, 6 or 9, but cube numbers can end in any number.



Do you agree with Tiny?

Tiny is correct about cube numbers, but square numbers can also end in zero, for example $10^2 = 100$

Multiply up to a 4-digit number by a 2-digit number

Notes and guidance

Building on their learning from previous years, children use long multiplication to multiply numbers with up to four digits by 2-digit numbers.

Children should already be aware that multiplication is commutative, so answers to calculations such as $56 \times 1,234$ can be found by rewriting as $1,234 \times 56$ and using the standard format.

Children also solve word problems and/or multi-step problems. This will be revisited in the next step, where alternative strategies are also explored, for example for multiplying by 9 or 99

Children who require additional support may benefit from revising multiplication of 2- or 3-digit numbers by a single digit before moving on to multiplication by a 2-digit number.

Things to look out for

- Children may omit the zero needed in the second line of a long multiplication.
- Children need to be secure with their times-tables, or have strategies for deriving them.
- When regrouping, children may misapply the procedure, particularly when a large number of digits are involved in the calculation.

Key questions

- How do you set out a long multiplication?
- Which number do you multiply by first?
- What is important to remember when you begin to multiply by the tens digit?
- When do you need to make an exchange? How do you do this?
- What happens if there is an exchange needed in the last step of the calculation?

Possible sentence stems

- To multiply by a 2-digit number, first multiply by the _____, then multiply by the _____ and then find the _____
- Multiplying by _____ is the same as multiplying by _____ and then multiplying the answer by _____

National Curriculum links

- Multiply multi-digit numbers up to four digits by a 2-digit whole number using the formal written method of long multiplication
- Solve problems involving addition, subtraction, multiplication and division

Multiply up to a 4-digit number by a 2-digit number

Key learning

- Work out 43×6
Use your answer to find the answer to 43×60

- Complete the calculations.

			2	3	
	×		6	4	
			9	2	
	+			0	

(23×4)
(23×60)

			3	1	2
	×			2	3
			9	3	6
	+				

(312×3)
(312×20)

- Work out the multiplications.

			4	2	6	7
	×				3	4

			3	0	4	6
	×				7	3

- 2,465 people buy tickets for a festival.
Each ticket costs £48
How much is spent altogether on the tickets?



- Work out the multiplications.

$$17 \times 562$$

$$23 \times 3,164$$

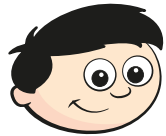
$$41 \times 5,312$$

- Huan receives a new comic book every month.
Each book has 36 pages.
He reads a comic book once a month for 6 years.
How many pages does Huan read altogether?
- There are 27 classes in a school.
There are 32 children in each class.
Can all the children in the school sit in a cinema with 1,000 seats?
If yes, how many spare seats will there be?
If no, how many more seats are needed?

Multiply up to a 4-digit number by a 2-digit number

Reasoning and problem solving

The product of a 4-digit number and a 2-digit number will always have at least six digits.



No

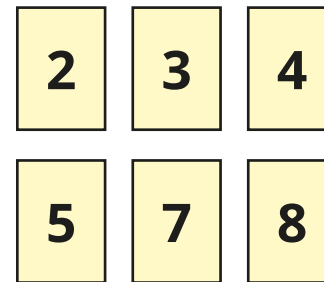
Do you agree with Dexter?
Explain your answer.



What is the product of the greatest 4-digit number and the greatest 2-digit number?



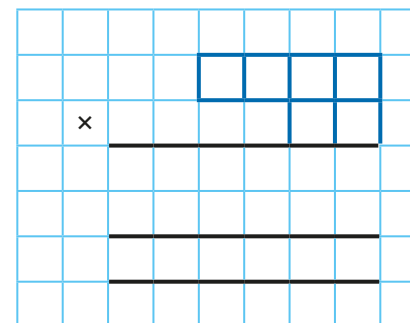
989,901



Write the digits in the boxes to find the greatest product.

You can use each digit once only.

$8,432 \times 75 = 632,400$



Solve problems with multiplication

Notes and guidance

In this small step, children use the column method for multiplication and explore alternative strategies for solving multiplication problems, including word problems.

Children use their knowledge of multiplying by powers of 10 and adjust calculations: for example, instead of multiplying a number by 99, they multiply the number by 100 and then subtract the number from the product.

Children explore using factors to find the answers to multiplication problems, such as multiplying by 5 and then by 7 as an alternative to multiplying by 35. This is a useful strategy for children who have good times-table knowledge but make errors with the algorithm for long multiplication.

Things to look out for

- Children may try to use formal methods when alternative strategies would be more appropriate.
- Children may need support to identify the most efficient method, for example $\times 100$ subtract $\times 1$ may be better than $\times 90$ add $\times 9$
- When using the factorisation method, children may forget to multiply the first product by the second factor.

Key questions

- What is the quickest way of multiplying whole numbers by 10/100/1,000?
- What number is 99 close to? How does this help you to multiply by 99?
- If you double a number and then double it again, what is the overall effect on the original number?
- What factor pairs have a product of _____? How does this help you to multiply by _____? Which factor pair is easiest to use?

Possible sentence stems

- To multiply by _____, I can multiply by _____ and add/subtract _____ to/from the product.
- _____ = _____ \times _____, so to multiply by _____ I can multiply by _____ and then multiply the product by _____

National Curriculum links

- Perform mental calculations, including with mixed operations and large numbers
- Solve problems involving addition, subtraction, multiplication and division

Solve problems with multiplication

Key learning

- Work out the multiplications.

78×10

63×100

$56 \times 1,000$

Use your answers to work out these multiplications.

78×9

63×99

56×999

- Office chairs cost £99

A company buys 38 chairs for its offices.

How much does the company pay altogether?

In a sale, the price of the chairs is reduced to £79

How much do 38 chairs cost at the sale price? How can you use your first answer to help you?

- Here is a strategy for multiplying numbers by 5

Multiply the number by 10 and find half of the answer.

Use the strategy to work out the multiplications.

84×5

628×5

$8,206 \times 5$

$3,512 \times 5$

Why does the strategy work?

- Explain why $83 \times 4 = 83 \times 2 \times 2$

Find the missing numbers.

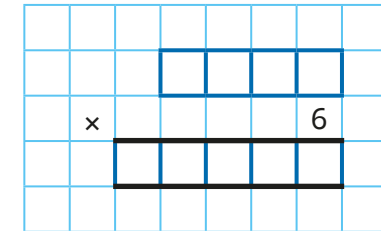
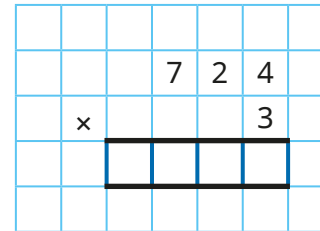
$37 \times 14 = 37 \times 2 \times \underline{\quad}$

$812 \times 25 = 812 \times 5 \times \underline{\quad}$

$256 \times 15 = 256 \times \underline{\quad} \times \underline{\quad}$

$902 \times 56 = \underline{\quad} \times \underline{\quad} \times 8$

- Complete the calculations to work out 724×18



Find a different way to work out 724×18

- Find the missing numbers.

$63 \times 24 = 63 \times 4 \times \underline{\quad}$

$63 \times 24 = 63 \times 3 \times \underline{\quad}$

Use both factorisations to work out 63×24

Which strategy did you find easier?

Use similar strategies to work out the multiplications.

84×15

326×45

612×42

$3,592 \times 32$

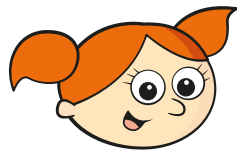
Solve problems with multiplication

Reasoning and problem solving

Alex is working out $6,412 \times 16$



I'm going to keep doubling 6,412 until I have found $6,412 \times 16$



How many calculations will Alex have to do?

Use Alex's method to find $6,412 \times 16$

How else could Alex multiply by 16?

Talk about it with a partner.



four calculations

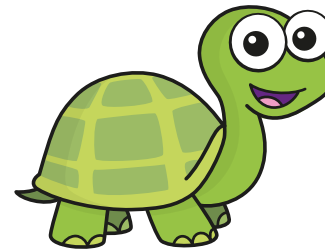
$$6,412 \times 2 = 12,824$$

$$6,412 \times 4 = 25,648$$

$$6,412 \times 8 = 51,296$$

$$6,412 \times 16 = 102,592$$

$35 = 1 \times 35$,
so I can work out
 832×35 by multiplying by 1
and then by multiplying
by 35



Explain why Tiny's strategy is not a good one.

Use a different factor pair of 35 to work out 832×35

Tiny's strategy is not good because you still have the same calculation of 832×35 after multiplying by 1

$$35 = 5 \times 7$$

$$832 \times 5 = 4,160 \text{ and } 4,160 \times 7 = 29,120$$

or

$$832 \times 7 = 5,824 \text{ and } 5,824 \times 5 = 29,120$$

Short division

Notes and guidance

In Year 5, children built on earlier learning of short division and learned to divide numbers with up to four digits by single-digit numbers. This small step reinforces all this earlier learning in preparation for the upcoming steps on long division.

Children perform short divisions both with integer answers and where there is a remainder. They interpret the remainder in context, for example knowing that “4 remainder 1” could mean 4 complete boxes with 1 left over so 5 boxes will be needed.

Children may need to list multiples of the number they are dividing by to help them if their times-table knowledge is not secure.

Things to look out for

- Children need to be confident with their times-tables “both ways”, i.e. knowing division facts as well as multiplication facts.
- Children may not recognise sharing and/or grouping division problems when presented in words.
- Numbers with placeholders (e.g. 80,320) may cause difficulty for children.
- Children may not be able to interpret the remainder.

Key questions

- How many groups of 4 _____ are there in 40/400/4,000?
- How many groups of 4 _____ are there in 80/800/8,000?
- What do you do with any remaining ones at the end of a division?
- If you cannot make a group in a column, what do you do?
- What does the remainder mean in this question?

Possible sentence stems

- _____ thousands divided by _____ is equal to _____ thousands with a remainder of _____
The remainder is exchanged into _____ hundreds.
- _____ hundreds divided by _____ is equal to _____ hundreds with a remainder of _____
The remainder is exchanged into _____ tens.

National Curriculum links

- Solve problems involving addition, subtraction, multiplication and division
- Divide numbers up to four digits by a 2-digit number using the formal written method of short division where appropriate, interpreting remainders according to the context

Short division

Key learning

- Work out the divisions mentally.

$8 \div 2$ $80 \div 2$ $800 \div 2$ $8,000 \div 2$
 $12 \div 4$ $120 \div 4$ $1,200 \div 4$ $1,200 \div 3$

- Complete the short divisions.

	3	9	6				

	3	6	4	2			

	3	5	1	2	7		

- Here is $8,524 \div 4$ shown using place value counters and short division.

Th	H	T	O
1,000 1,000 1,000 1,000	100 100 100 100	10 10 10 10	1 1 1 1
1,000 1,000 1,000 1,000	100	10 10 10 10 10 10	

		2	1	3	1		
	4	8	5	2	4		

Use this method to work out the divisions.

$5,520 \div 4$	$6,432 \div 3$	$2,665 \div 5$
----------------	----------------	----------------

- Complete the short divisions.

	3	8	6				

	5	6	7	3			

	4	5	3	2	2		

- 1,480 pencils are grouped into packets of 5
How many groups of 5 pencils are there?



- 650 children from a school go to a theme park.
On the first ride, each car seats 4 children.

How many cars are needed for the whole school to go on the first ride?

On the second ride, each car seats 6 children.

How many cars are needed for the whole school to go on the second ride?

- Tickets to see the school play cost £9

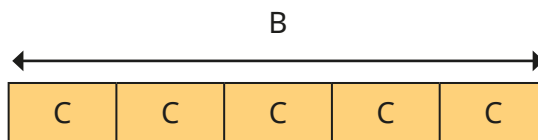
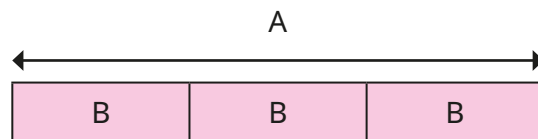
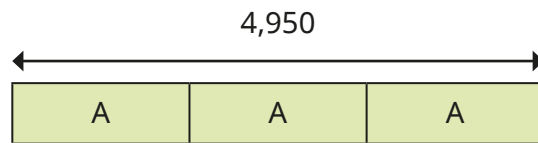
How many tickets can be bought with £100?

How many tickets can be bought with £350?

Short division

Reasoning and problem solving

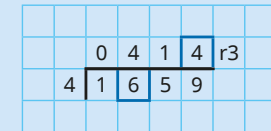
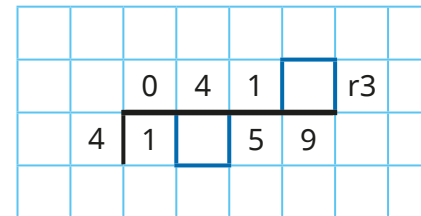
Here are three bar models.
They are not drawn to scale.



Work out the value of C.

A = 1,650
B = 550
C = 110

Work out the missing digits.



Work out the divisions.

$$275 \div 11$$

$$3,366 \div 11$$

$$6,036 \div 12$$

$$2,356 \div 12$$

25
306
503
196 r4

Compare methods with a partner.



Division using factors

Notes and guidance

In this small step, children build on their understanding of using factors in multiplication and learn to divide by a 2-digit number using repeated division.

Children start with the familiar strategy that to divide by 4 they can halve and halve again. They move on to dividing by multiples of 10 before looking at slightly more complex divisions using two single-digit factors. It may be worth revising what factor pairs are and practising finding factor pairs of 2-digit numbers. Children need to be aware that the divisions can be carried out in any order. This means they can choose to divide first by the factor they find it easier to work with, and then by the factor they find more difficult.

Things to look out for

- Children may partition the number they are dividing by into tens and ones instead of using factors.
- Children may factorise the number they are dividing by incorrectly.
- Children may need support identifying the most efficient pair of factors to use.
- Children may identify 1 and the number itself as a pair of factors and should recognise that this does not simplify the calculation.

Key questions

- What does the word “factor” mean?
- What are the factors of the number you are dividing by?
- What numbers do you find it easy to divide by?
- How can you check your answer?
- Which factor are you going to divide by first/second? Why?

Possible sentence stems

- Dividing by 4 is the same as dividing by _____ and _____ again.
- The factor pairs of _____ are _____
- To divide by _____, I can first divide by _____ and then divide the answer by _____
- _____ = _____ × _____, so to divide by _____ I can divide by _____ and then divide the answer by _____

National Curriculum links

- Solve problems involving addition, subtraction, multiplication and division

Division using factors

Key learning

- Take 20 counters and share them into two equal groups.

Share each of these groups into two equal groups.

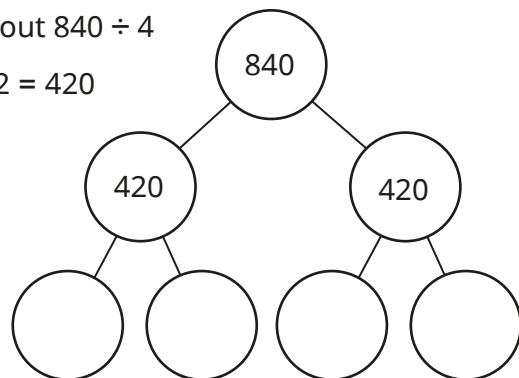
How many groups have you got now?

Complete the calculation.

$$20 \div 2 \div 2 = 20 \div \underline{\quad} = \underline{\quad}$$

- Esther is working out $840 \div 4$

She knows $840 \div 2 = 420$



How can Esther use this fact to help find $840 \div 4$?

- 80 counters are divided into 10 equal groups.

How many counters are there in each group?

The counters are then shared into 2 equal groups.

How many counters are there in each group now?

- Complete the calculations.

▶ $600 \div 30 = 600 \div 10 \div \underline{\quad} = 60 \div \underline{\quad} = \underline{\quad}$

▶ $600 \div 20 = 600 \div 10 \div \underline{\quad} = 60 \div \underline{\quad} = \underline{\quad}$

▶ $600 \div 40 = 600 \div 10 \div \underline{\quad} = 60 \div \underline{\quad} = \underline{\quad}$

- Work out the divisions.

$$900 \div 30$$

$$640 \div 40$$

$$650 \div 50$$

$$540 \div 20$$

- Find $720 \div 15$ by firstly dividing 720 by 5 and then dividing the result by 3

Why does dividing a number by 5 and then dividing by 3 give you the same answer as dividing the number by 15?

Use this strategy to work out the divisions.

$$570 \div 15$$

$$560 \div 14$$

$$720 \div 18$$

$$725 \div 25$$


$$560 \div 14$$

$$1,176 \div 24$$

Can any of the divisions be done in more than one way?

Division using factors

Reasoning and problem solving



To calculate $4,320 \div 15$, I will first divide 4,320 by 5 and then divide the answer by 10

Explain why Tommy is wrong.

Tommy has partitioned 15 into $5 + 10$ instead of using the factor pair $3 \times 5 = 15$

Dividing by 5 and then dividing by 10 is the same as dividing by 50

Use factor pairs to work out the divisions.

$1,248 \div 48$


$1,248 \div 24$

$1,248 \div 12$

What do you notice about your answers?

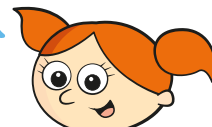
26, 52, 104

When the number you are dividing by is halved, the answer is doubled.




I'm going to work out $4,632 \div 12$ by dividing 4,632 by 3 and then dividing the result by another number.

Annie



I'm going to work out $4,632 \div 12$ by dividing 4,632 by 2 and then dividing the result by another number.

Alex



I'm going to work out $4,632 \div 12$ using short division.

Amir

Compare the children's methods.

Children should compare the methods while also recognising that each child gets the same answer.

Introduction to long division

Notes and guidance

In this small step, children are introduced to long division as a different method for dividing by a 2-digit number, now including numbers that cannot be factorised into single-digit numbers.

Children divide 3-digit numbers without remainders, using an expanded method that shows the multiples, before progressing to a more formal long division method. They divide 4-digit numbers, still without remainders, using their knowledge of multiplying by 10 and 100. When dividing by composite numbers, it may be worth comparing the long division method with the method of division using factors covered in the previous small step.

Long division with remainders is covered in the next small step.

Things to look out for

- Children may need support in setting out the long divisions, for example by providing the questions on pre-prepared squared grids with the questions already formatted.
- When dividing by prime numbers or large numbers, children may need support in working out the multiples of the number they are dividing by.

Key questions

- How can you use multiples to divide by a 2-digit number?
- Why do we subtract as we go along?
- What does the arrow represent in the long division?
- Can this division be done using factors instead? Why or why not?
- What is the first step when performing a long division?

Possible sentence stems

- _____ hundreds divided by _____ is equal to _____ hundreds with a remainder of _____
The remainder is exchanged into _____ tens.
- _____ tens divided by _____ is equal to _____ with a remainder of _____
The remainder is exchanged into _____ ones.

National Curriculum links

- Divide numbers up to four digits by a 2-digit whole number using the formal written method of long division, and interpret remainders as whole number remainders, fractions, or by rounding, as appropriate for the context
- Solve problems involving addition, subtraction, multiplication and division

Introduction to long division

Key learning

- Here is $360 \div 12$ using the long division method.

		0	3	6	
12		4	3	2	
		3	6	0	
			7	2	
			7	2	
				0	

(12 × 30)

(12 × 6)

Multiples of 12: $12 \times 1 = 12$

$12 \times 2 = 24$

$12 \times 3 = 36$

$12 \times 4 = 48$

$12 \times 5 = 60$

$12 \times 6 = 72$

Use this method to work out the divisions.

750 ÷ 15	765 ÷ 17	702 ÷ 18
----------	----------	----------

- Here is a different way of setting out a long division.

		0	3	6	
12		4	3	2	
		3	6		
			7	2	
			7	2	
				0	

Use this method to work out the divisions.

836 ÷ 11	798 ÷ 14	608 ÷ 19
----------	----------	----------

- Here is $7,355 \div 15$ using the long division method.

		0	4	8	9
15		7	3	3	5
		6	0	0	0
			1	3	3
			1	2	0
				1	3
				1	3
					0

(15 × 400)

(15 × 80)

(15 × 9)

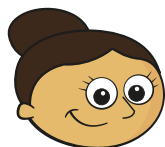
Use this method to work out the divisions.

2,208 ÷ 16	1,755 ÷ 45	1,536 ÷ 16
------------	------------	------------

- There are 1,989 players in a football tournament. Each team has 11 players and 2 reserves. How many teams are playing in the tournament?
- A farmer packs 8,280 eggs into cartons of 24. Use long division to find the number of cartons needed. Check your answer by dividing by factors.

Introduction to long division

Reasoning and problem solving



Dora

I'm going to work out $6,756 \div 12$ by dividing 6,756 by 3 and then dividing the result by 4



Mo

I'm going to work out $6,756 \div 12$ using long division.



Jack

I'm going to work out $6,756 \div 12$ using short division.

Compare the children's methods and talk about your favourite with a partner.

Children should recognise that each child gets the same answer despite using different methods.

$$6,120 \div 17 = 360$$

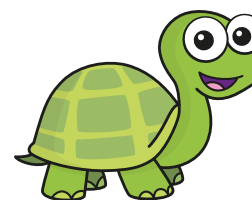


Use the given calculation to work out the missing number.

$$6,480 \div \underline{\hspace{2cm}} = 360$$

18

$1,950 \div 13$ is greater than $1,950 \div 15$



Tiny is correct.

Find how much greater $1,950 \div 13$ is than $1,950 \div 15$

$1,950 \div 13$ is 20 greater than $1,950 \div 15$

Long division with remainders

Notes and guidance

Now that children have learned to use the algorithm for long division with integer answers, they move on to long divisions with remainders.

This small step includes context questions where children interpret the remainder and/or adjust the number they are dividing. For example, when thinking about packing items into boxes, they consider the number of full boxes or the total number of boxes needed.

Children should always check that the remainder is less than the number they are dividing by. They can use estimation as a sense-check for their answers, for example $834 \div 18$ is close to $800 \div 20$ so the answer should be in the region of 40

Things to look out for

- Children may need support in setting out the long divisions, for example by providing the questions on pre-prepared squared grids with the questions already formatted.
- When dividing by prime numbers or large numbers, children may need support in working out the multiples of the number they are dividing by.

Key questions

- Why do we subtract as we go along?
- In a long division, what happens after the subtractions if you cannot divide exactly?
- What is the first step when performing a long division?

Possible sentence stems

- _____ hundreds divided by _____ is equal to _____ hundreds with a remainder of _____
The remainder is exchanged for _____ tens.
- _____ cannot be divided by _____, so there is a _____ of _____

National Curriculum links

- Divide numbers up to four digits by a 2-digit whole number using the formal written method of long division, and interpret remainders as whole number remainders, fractions, or by rounding, as appropriate for the context
- Solve problems involving addition, subtraction, multiplication and division

Long division with remainders

Key learning

- Filip uses multiples to help divide 372 by 15

		0	2	4	r	12
15		3	7	2		
		3	0	0		
			7	2		
			6	0		
			1	2		

Multiples of 15: $15 \times 1 = 15$
 $15 \times 2 = 30$
 $15 \times 3 = 45$
 $15 \times 4 = 60$

(15 × 20)
 (15 × 4)

Use Filip's method to work out the divisions.

$271 \div 17$	$623 \div 21$	$842 \div 32$
---------------	---------------	---------------

- Here is Aisha's method for finding 1,426 divided by 13

		0	1	0	9	r	9
13		1	4	2	6		
		1	3	0			
			1	2	6		
			1	1	7		
					9		

Use Aisha's method to work out the divisions.

$2,637 \div 16$	$4,453 \div 22$	$4,203 \div 18$
-----------------	-----------------	-----------------

- Mrs Hall needs 380 cupcakes for a party.



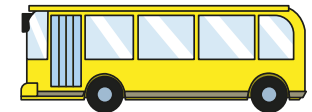
Cupcakes are sold in boxes of 15
 How many boxes of cupcakes does she need to buy?
 Will she have any cupcakes spare?
 How do you know?

- One day, a bakery produces 7,849 biscuits.



The biscuits are packed into boxes of 64 biscuits.
 How many full boxes can be packed?

- 576 children and 32 adults need transport for a school trip.



A coach has seats for 55 people.
 How many coaches are needed?
 How many spare seats will there be?

- A portion of rice is 65 g.



How many portions can be served from an 8 kg bag of rice?
 Will there be any rice left over?
 If yes, how much?

Long division with remainders

Reasoning and problem solving

Which calculations will definitely have a remainder?



A $8,164 \div 20$

B $7,836 \div 15$

C $4,678 \div 18$

D $6,751 \div 12$

How do you know?



All the calculations will have a remainder.

Two digits are missing from the division.



					r	14	
18	6						

The missing digits are equal.

What must they be?

What could the digits be if they were not equal?

4 and 4

2 and 6

6 and 2

8 and 0

9 and 8

$835 \div 17 = 48 \text{ r}19$

Explain why the calculation cannot be correct.

The remainder cannot be greater than 17

Solve problems with division

Notes and guidance

In this small step, children explore division problems, looking at the most appropriate strategy for finding a solution.

As well as providing an opportunity to revisit the learning of the last few steps, children look at alternative methods such as partitioning the number into appropriate multiples of the number they are dividing by. They also use counting up in multiples, for example for calculations such as $1,400 \div 200$, and compare this with other strategies.

Encourage children to think about the numbers in a division question and to consider alternative strategies before they launch into a formal method.

Later in this block, children explore using known division facts to find other division or multiplication facts.

Things to look out for

- Children may try to use formal methods when alternative strategies would be more appropriate.
- Children may try to apply strategies that work for multiplication to division situations where they do not work.
- Interpreting remainders in a given context can be challenging for children.

Key questions

- What is the most useful way of partitioning the number?
- Would you use short division or long division? Why?
- If you double a number and then double it again, what is the overall effect on the original number?
- What factor pairs have a product of _____? How does this help you to divide by _____? Which factor pair is easiest to use?

Possible sentence stems

- I will partition the number into _____ and _____ because both _____ and _____ are divisible by _____
- _____ = _____ \times _____, so to divide by _____ I can divide by _____ and then divide the quotient by _____

National Curriculum links

- Perform mental calculations, including with mixed operations and large numbers
- Solve problems involving addition, subtraction, multiplication and division

Solve problems with division

Key learning

- Complete the workings for $560 \div 4$

$$400 \div 4 = \underline{\quad}$$

$$160 \div 4 = \underline{\quad}$$

So $560 \div 4 = \underline{\quad} + \underline{\quad} = \underline{\quad}$

- Use partitioning to work out the divisions.

$861 \div 41$	$102 \div 6$	$1,236 \div 12$

- Which of the divisions can you work out mentally?

$340 \div 10$	$608 \div 2$	$500 \div 20$
$631 \div 1$	$2,100 \div 700$	$432 \div 18$

- Use your preferred method to work out the divisions.

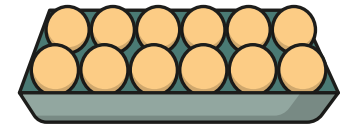
$780 \div 30$	$824 \div 4$	$900 \div 30$
$1,197 \div 21$	$4,200 \div 21$	$1,110 \div 15$

Did you use the same method for each question?

- Tom has saved £8 in 20p coins.
How many 20p coins does Tom have?



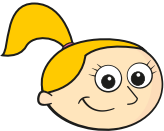
- Eggs are packed in trays of 12
The trays are packed into boxes.
Each box contains 480 eggs.
How many trays are in each box?




- A builder needs 8,600 bricks to build a wall.
There are 800 bricks in a load.
How many loads must the builder buy?

Solve problems with division

Reasoning and problem solving




To divide a number by 5, I can divide the number by 10 and then halve the answer.



Eva

To divide a number by 5, I can divide the number by 10 and then double the answer.



Ron

Who is correct?

Why is the other person incorrect?

Use the correct strategy to work out the divisions.


$2,000 \div 5$	$3,600 \div 5$
$310 \div 5$	$100,000 \div 5$

Ron

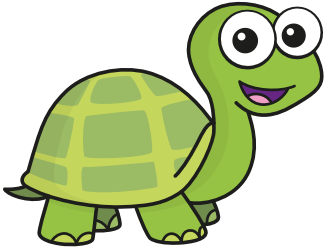
Eva's strategy will give the result for the number divided by 20

400, 720

62, 20,000

Tiny is trying to divide by 9 

$10 - 1 = 9$, so to divide by 9, I need to divide by 10 and subtract the number again.



Explain why Tiny is wrong.

Tiny is confusing strategies for multiplication and division.

Solve multi-step problems

Notes and guidance

In this small step, children apply the skills they have developed so far in this block to solving problems in real-life contexts.

The problems involve more than one calculation and children must decide which operations they need to perform and in what order to perform them; this will need careful modelling. As the focus of the step is making the correct choice of operation, calculators can be provided or the numbers simplified if necessary. Children should be encouraged to think about the best way to perform any of the calculations and use the most appropriate written, informal or mental method. For example, this might include using a number line to work out a subtraction after a long multiplication.

Things to look out for

- In longer problems, children may find the number of words overwhelming and need encouragement to split the problem down into smaller parts.
- Children may find choosing the correct operation difficult.
- Children may need support to set out solutions with several parts clearly.

Key questions

- What can you work out first?
- Is this step an addition, a subtraction, a multiplication or a division? How can you tell?
- Could you draw a diagram to represent the problem?
- Can you work out the answer to this part of the problem mentally or do you need another method?
- What can you do next?

Possible sentence stems

- First, I need to work out _____
The calculation I need to do is _____
- Next, I need to work out _____
The calculation I need to do is _____

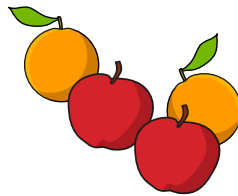
National Curriculum links

- Solve addition and subtraction multi-step problems in contexts, deciding which operations and methods to use and why
- Solve problems involving addition, subtraction, multiplication and division

Solve multi-step problems

Key learning

- The total mass of apples in a box is 25 kg.
The total mass of oranges in a box is 24 kg.
 - ▶ There are 32 boxes of apples and 25 boxes of oranges in a supermarket.
What is the total mass of apples and oranges?
 - ▶ A customer orders 300 kg of apples and 600 kg of oranges.
How many boxes of fruit will the customer receive?



- There are 80 g of pasta in one portion.
How much pasta is needed for 12 portions?
How many portions can be made from a 16 kg bag of pasta?

- At a parade, there are 25 rows of people with 8 people in each row.
Each person holds 2 flags.
How many flags are needed for the parade?



- A coach has 55 seats and a minibus has 17 seats.
431 people from a school go on a trip.
The school books 6 coaches and 8 minibuses.
How many spare seats will there be?

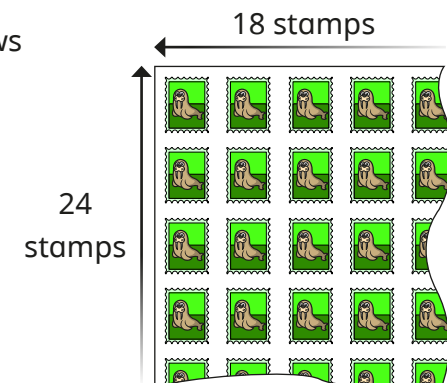
- Five boxes of toy trains cost £120.
Each box contains 6 trains.
How much does each train cost?



- Dr Patel can type 40 words a minute.
How many words can she type in an hour?
How long does it take Dr Patel to type 1,000 words?

- A headteacher has £2,000 to spend on new furniture.
He wants to buy 15 desks for £79 each and 30 chairs for £29 each.
Does he have enough money?

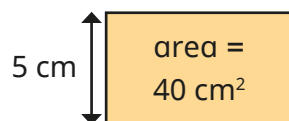
- A sheet of stamps has 24 rows and 18 columns of stamps.
How many stamps are there altogether on 35 sheets?



Solve multi-step problems

Reasoning and problem solving

The area of a rectangular tile is 40 cm^2
The width of the tile is 5 cm.



A strip of tiles is made by laying tiles end-to-end.



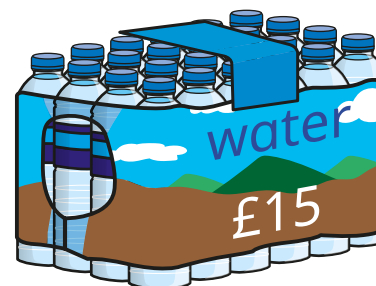
How long is a strip with 15 tiles?
How many tiles are needed to make a strip 280 cm long?
How many tiles are needed to make a strip 4 m long?

120 cm

35 tiles

50 tiles

24 bottles of water cost £15



How many bottles of water can you buy for £30?
How many bottles of water can you buy for £300?
How many bottles of water can you buy for £525?
How much will 600 bottles of water cost?

48 bottles

480 bottles

840 bottles

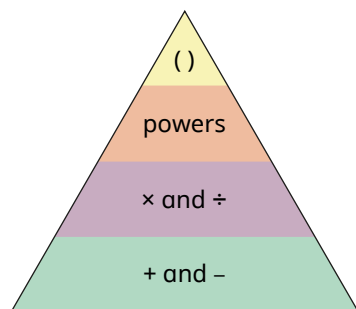
£375

Order of operations

Notes and guidance

In this small step, children learn the order of priority for operations in a calculation: that calculations in brackets should always be done first, and that multiplication and division have equal priority and should be performed before additions and subtractions.

This image may be useful when teaching the order of operations.



Things to look out for

- If children have heard acronyms such as BIDMAS or BODMAS, they may mistakenly think that addition should be done before subtraction and incorrectly work out, for example, $10 - 3 + 4$ as $10 - 7 = 3$
- Similarly, children may not be aware that multiplication and division are of equal priority.

Key questions

- Does it make a difference if you perform the operations in a different order?
- What do brackets in a calculation mean? What would happen if you did not use the brackets?
- Which operation has greater priority, addition or multiplication?
- How many pairs of operations do you know that have equal priority?
- How do you find the square of a number?

Possible sentence stems

- _____ has greater priority than _____, so the first part of the calculation I need to do is _____

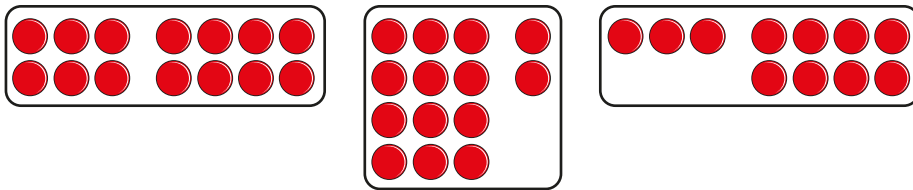
National Curriculum links

- Perform mental calculations, including with mixed operations and large numbers
- Use their knowledge of the order of operations to carry out calculations involving the four operations

Order of operations

Key learning

- Match the counters to the calculations.



$3 + 4 \times 2$	$3 \times 4 + 2$	$(3 + 4) \times 2$
------------------	------------------	--------------------

- Draw counters to represent each calculation.

$4 + 1 \times 3$	$(4 + 1) \times 3$
------------------	--------------------

Work out the answers.

- Work out the calculations.

$(5 + 2) \times 3$	$6 + 4 \div 2$	$10 - 4 \div 2$
$5 + 2 \times 3$	$(6 + 4) \div 2$	$(10 - 4) \div 2$

- Add brackets to make the calculations correct.

▶ $6 + 4 \times 3 = 30$	▶ $20 - 20 \times 2 = 0$
▶ $12 \times 3 - 1 = 24$	▶ $10 \div 2 + 3 = 2$

- Work out the calculations.

$6 \times 4 + 5 \times 2$	$6 \times 4 - 5 \times 2$	$6 \times (4 + 5) \times 2$
---------------------------	---------------------------	-----------------------------

- Dani has 7 bags with 5 sweets in each bag. She adds one more sweet to each bag.

Which calculation shows how many sweets there are in total?

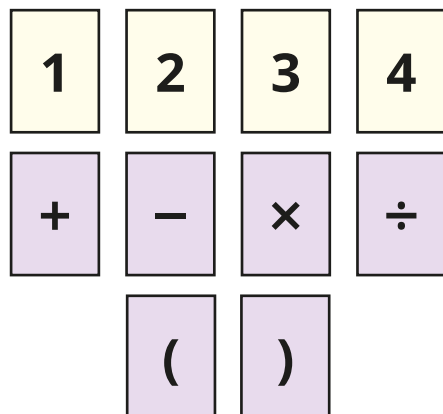
$7 \times (5 + 1)$	$7 \times 5 + 1$
--------------------	------------------

- Work out the calculations.

$6^2 - 3 \times 4$	$6^2 \div (4 + 5)$	$(7 - 4)^2$
--------------------	--------------------	-------------

Order of operations

Reasoning and problem solving



Use the digits and symbols to write as many calculations as you can that give different answers.

Is it possible to make every number from zero to 20?

multiple possible answers, e.g.

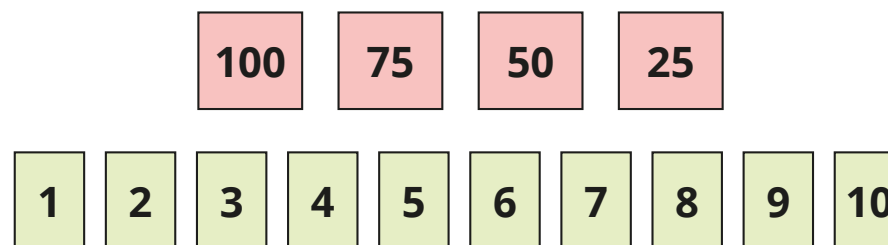
$$1 \times 2 \times 3 + 4 = 10$$

$$(1 + 2) \times 3 + 4 = 13$$

$$(1 + 2) \times (3 + 4) = 21$$

$$(1 + 2 + 3) \times 4 = 24$$

Here are some number cards.



Pick **one** large number from the top row.

Pick **five** smaller numbers from the bottom row.

Use a calculator or computer to generate a 3-digit target number.

Use your numbers, the four operations and brackets to find a number as close as possible to the target number.

Compare answers as a class.

Mental calculations and estimation

Notes and guidance

Children should use mental strategies and estimation whenever appropriate, and several examples have been included throughout the block. This small step reminds children of the importance of mental strategies and estimation, and gives them an opportunity to revisit and extend their learning from this block and previous years.

Children should be aware that estimating the answer of a calculation serves as a sense-check on whether their answer is correct, and this can be done either before or after a calculation. The numbers they choose when performing estimates should be simple enough for this to be done mentally.

Links should be made back to previous learning on rounding when simplifying numbers within a calculation.

Things to look out for

- Children may try to use formal methods when alternative strategies would be more appropriate.
- Children may not round numbers to an appropriate degree of accuracy. For example, 4-digit numbers should usually be rounded to the nearest 1,000 and not to the nearest 100 or nearest 10

Key questions

- Should you round the number to the nearest 10/100/1,000? Why?
- Are any of the numbers multiples of powers of 10? How does this help you to add/subtract/multiply/divide the numbers?
- What number is (for example) 99 close to? How does this help with the calculation? What adjustment do you need to make?
- How would partitioning/reordering the number(s) help?
- Why are estimates to the answers of calculations useful?

Possible sentence stems

- The previous multiple of _____ is _____
- The next multiple of _____ is _____
- _____ rounded to the nearest _____ is _____

National Curriculum links

- Use estimation to check answers to calculations and determine, in the context of a problem, an appropriate degree of accuracy
- Perform mental calculations, including with mixed operations and large numbers

Mental calculations and estimation

Key learning

- Use rounding to estimate the answer to each calculation.

$6,941 + 4,099$

$6,941 - 4,099$

$6,941 \times 18$

$6,941 \div 11$

Compare answers with a partner.

- What strategies would you use to find the exact answers to the calculations?

$480 + 20$

$480 - 20$

480×20

$480 \div 20$

Compare answers with a partner.

- How could you change the order of the numbers in each of the calculations to make them easier to do mentally?

$97 + 58 + 43$

$68 + 57 - 28$

$12 \times 9 \times 5$

$50 \times 16 \times 2$

$4 \times 17 \times 25$

Work out the answers to the calculations.

- It is 816 km from Mr Trent's house to Glasgow.
He drives 583 km of the way.



Approximately how much further does he have to drive?

- A textbook costs £19.99
Approximately how many textbooks can be bought for £300?

- Work out the calculations.

$736 + 99$

$12,000 - 3$

$8,567 - 999$

56×9

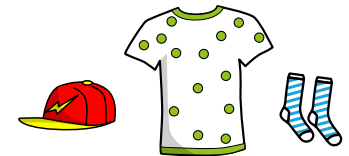
$6,999 + 8,500$

34×20

$8,000 \div 20$

$8,204 - 6,899$

- Mo wants to buy a T-shirt for £9.99,
a pair of socks for £2.49
and a cap for £8.99



He has £22 in his wallet.

How can he quickly check whether he has enough money?

Mental calculations and estimation

Reasoning and problem solving

Here is a number line.



Estimate the number shown by arrow B for these values of A and C:

- A = 0 and C = 1,000
- A = 30 and C = 230
- A = 7 and C = 33
- A = 1 and C = 2
- A = 1,000 and C = 100,000

B is approximately nine-tenths of the way from A to C, so answers should be around:

- 900
- 210
- 30
- 1.9
- 90,000

$$2,000 - 1,287$$

Here are three strategies for working out the subtraction.



Whitney

I will use the column method.



Dexter

I will use number bonds from 87 to 100, then from 1,300 to 2,000



Teddy

I will subtract one from each number and then use the column method.

Whose strategy is most efficient?

Children can choose any strategy with the correct justification.

Reason from known facts

Notes and guidance

In this small step, children work out other facts from a given fact using their knowledge of place value, inverse operations, commutativity and the mental strategies practised in this block, particularly in the previous small step. Using diagrams, including area models and number lines, can help children to see the links between the different calculations. They need to be confident in multiplying and dividing by powers of 10. Children also explore the idea of doubling and halving.

It is important that children can not only work out an answer of a related fact, but also explain the connections between calculations that helped them arrive at this answer.

This small step will focus on integers, and decimal calculations will be covered in Spring Block 3

Things to look out for

- Children may try to calculate the answers instead of looking at the relationships between the calculations and using reasoning.
- Children may over-generalise and try to use multiplication strategies that do not work for other operations.
- Children may need support to see the connections between the given fact and the adjusted calculation.

Key questions

- What is an inverse operation?
- How can you use an inverse operation to find related facts?
- What is the same and what is different about the numbers in the given calculation and the numbers in the calculation you want to work out?
- How will the answer change if you increase/decrease/multiply/divide one/both of the numbers by _____?

Possible sentence stems

- If I add/subtract _____ to/from one of the numbers in the calculation, then the answer will change by _____
- If I multiply/divide _____ one of the numbers in the calculation by _____, then the answer will change by _____

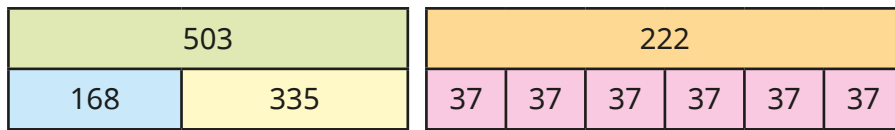
National Curriculum links

- Perform mental calculations, including with mixed operations and large numbers
- Solve problems involving addition, subtraction, multiplication and division

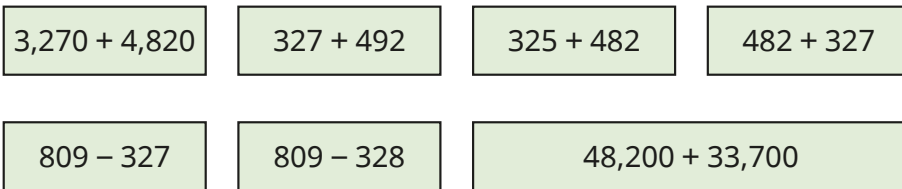
Reason from known facts

Key learning

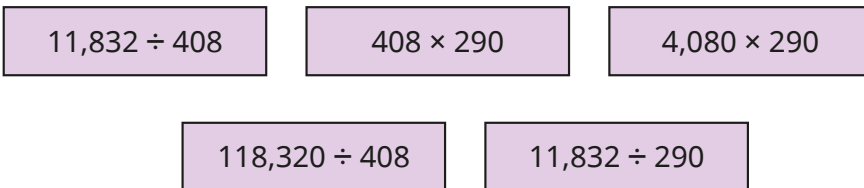
- Write four facts shown by each bar model.



- Use the fact that $327 + 482 = 809$ to work out the answers to the calculations.



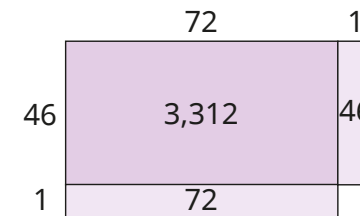
- Use the fact that $11,832 \div 29 = 408$ to work out the answers to the calculations.



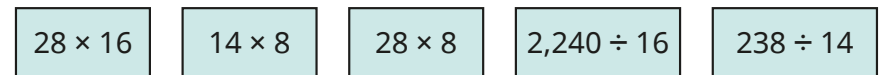
- Use the fact that $46 \times 72 = 3,312$ to work out the multiplications.



You can use the area model to help you.



- Use the fact that $5,138 \div 14 = 367$ to work out 15×367
- Use the fact that $14 \times 16 = 224$ to work out the calculations.



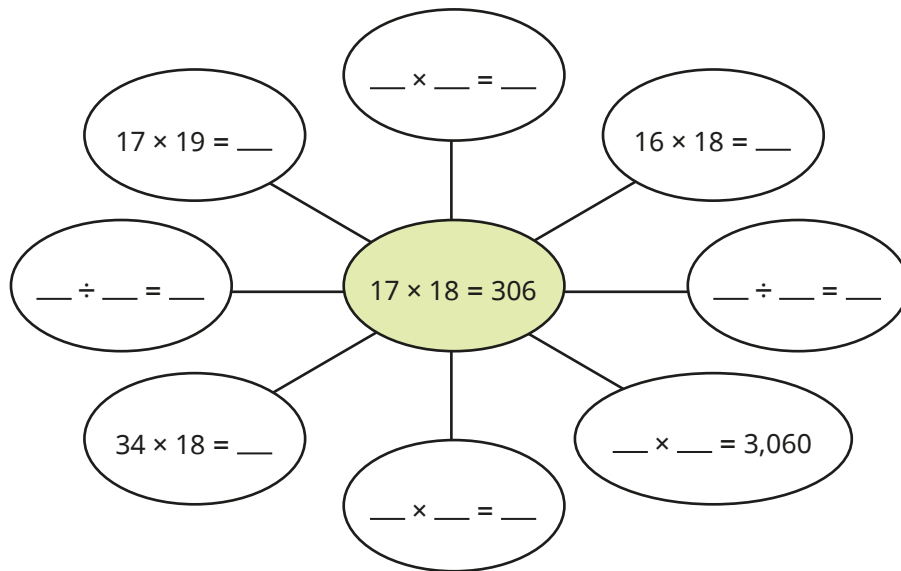
- Work out the missing numbers.

- $537 + 464 = 470 + \underline{\hspace{2cm}}$
- $25 \times 30 = 50 \times \underline{\hspace{2cm}}$
- $942 - 199 = \underline{\hspace{2cm}} - 200$
- $980 \div 20 = 1,000 \div 20 - \underline{\hspace{2cm}}$
- $38 \times 80 = 160 \times \underline{\hspace{2cm}}$
- $45 \times 79 = 45 \times \underline{\hspace{2cm}} - 45$

Reason from known facts

Reasoning and problem solving

Complete the spider diagram.



Compare methods with a partner.



$17 \times 19 = 323$

$34 \times 18 = 612$

$16 \times 18 = 288$

$170 \times 18 \text{ or } 17 \times 180 = 3,060$

Without working them out, which calculation has the greater answer?

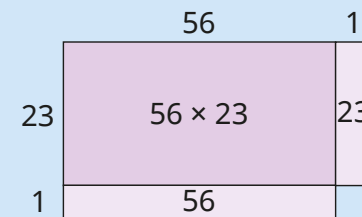


57×23

56×24

Draw a diagram to explain how you know.

Compare both calculations to 56×23



56×24 is 56 greater than 56×23

57×23 is only 23 greater than 56×23

So 56×24 is greater.