

Autumn Block 3

Fractions A

Small steps

Step 1

Equivalent fractions and simplifying

Step 2

Equivalent fractions on a number line

Step 3

Compare and order (denominator)

Step 4

Compare and order (numerator)

Step 5

Add and subtract simple fractions

Step 6

Add and subtract any two fractions

Step 7

Add mixed numbers

Step 8

Subtract mixed numbers

Small steps

Step 9

Multi-step problems

Equivalent fractions and simplifying

Notes and guidance

In this small step, children build on prior knowledge of equivalent fractions to recognise when fractions are, and are not, in their simplest form.

Children use their understanding of common factors to simplify fractions. They learn that when the numerator and denominator have no common factors greater than 1, the fraction is in its simplest form.

The step begins with fractions with one common factor (greater than 1) and moves on to fractions with several common factors. Children are encouraged to look for the greatest possible number to divide by, but also understand that simplification can be performed in more than one step.

Pictorial representations and fraction walls can be used to support understanding.

Things to look out for

- Children may partially simplify a fraction instead of finding the simplest form, for example $\frac{6}{24} = \frac{3}{12}$
- When simplifying mixed numbers, children may divide the whole number as well as the numerator and denominator.

Key questions

- What are the common factors of _____ and _____?
- Why is it better to identify the greatest possible number that both the numerator and denominator can be divided by?
- Does the simplified fraction have the same value?
- Do the numerator and denominator have any more common factors?
- How can you tell if a fraction is in its simplest form?
- When simplifying a mixed number, why does the integer not change?

Possible sentence stems

- Both the numerator and the denominator can be divided by _____
- To simplify the fraction, I will divide the numerator and denominator by _____
- _____ in its simplest form is _____

National Curriculum links

- Use common factors to simplify fractions; use common multiples to express fractions in the same denomination

Equivalent fractions and simplifying

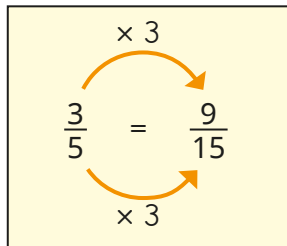
Key learning

- Here are some fractions.

$\frac{4}{5}$	$\frac{30}{60}$	$\frac{7}{8}$	$\frac{42}{48}$	$\frac{2}{6}$	$\frac{1}{2}$	$\frac{8}{10}$	$\frac{16}{48}$
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Find the pairs of equivalent fractions.

- Jack uses multiplication to find equivalent fractions.



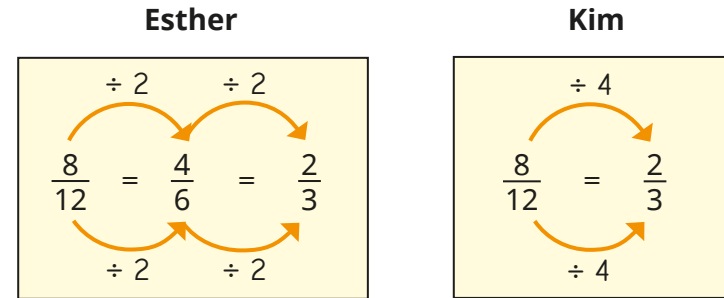
Use Jack's method to complete the equivalent fractions.

$\frac{4}{5} = \frac{\square}{20}$
 $\frac{4}{5} = \frac{20}{\square}$
 $\frac{\square}{7} = \frac{9}{21}$
 $\frac{4}{7} = \frac{\square}{21}$

- Use division to write the fractions in their simplest form.

$\frac{12}{15} = \frac{4}{\square}$
 $\frac{12}{20} = \frac{\square}{5}$
 $\frac{16}{24} = \frac{2}{\square}$
 $\frac{10}{12} = \frac{\square}{\square}$
 $\frac{6}{30} = \frac{\square}{\square}$
 $\frac{24}{40} = \frac{\square}{\square}$

- Esther and Kim are simplifying fractions.



What is the same? What is different?

Use one of their methods to simplify the fractions.

$\frac{2}{12}$	$\frac{4}{12}$	$\frac{6}{12}$	$\frac{6}{24}$	$\frac{8}{24}$	$\frac{16}{24}$
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- Mo is simplifying $2\frac{4}{10}$



To simplify $2\frac{4}{10}$, keep the whole number the same and simplify the fraction.

$$2\frac{4}{10} = 2\frac{2}{5}$$

Use Mo's method to simplify the mixed numbers.

$3\frac{4}{10}$	$4\frac{12}{20}$	$6\frac{16}{30}$	$2\frac{16}{40}$
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Equivalent fractions and simplifying

Reasoning and problem solving

Tom and Aisha are simplifying an improper fraction.

Tom

Aisha

$$\frac{36}{8} = 4\frac{4}{8} = 4\frac{1}{2}$$

$$\frac{36}{8} = \frac{9}{2} = 4\frac{1}{2}$$

various answers

Whose method do you prefer?

Explain your answer.



Tiny is simplifying $4\frac{12}{16}$

$$4\frac{12}{16} = 1\frac{3}{4}$$

Explain Tiny's mistake.



Tiny has divided the whole number by 4 instead of just simplifying the fraction.

Here are some fractions.



$$\frac{5}{15}$$

$$\frac{2}{4}$$

$$\frac{4}{16}$$

$$\frac{8}{16}$$

$$\frac{5}{10}$$

$$\frac{3}{9}$$

$$\frac{6}{12}$$

$$\frac{2}{8}$$

Which of the fractions:

- simplify to $\frac{1}{2}$
- simplify to $\frac{1}{3}$
- simplify to $\frac{1}{4}$?

What patterns can you see?

What is the relationship between the numerator and the denominator?

Identify three more fractions that could go in each list.



simplifies to $\frac{1}{2}$:

$$\frac{2}{4}, \frac{8}{16}, \frac{5}{10}, \frac{6}{12}$$

simplifies to $\frac{1}{3}$:

$$\frac{5}{15}, \frac{3}{9}$$

simplifies to $\frac{1}{4}$:

$$\frac{4}{16}, \frac{2}{8}$$

multiple possible answers

Equivalent fractions on a number line

Notes and guidance

In this small step, children use number lines to count forwards and backwards in fractions and to find equivalent fractions.

Children start by revising counting fractions above 1 on a number line to ensure they are able to count in fractions accurately. Using a number line clearly shows that finding equivalent fractions does not change the value of the fraction. Encourage children to draw extra intervals on number lines to support them in placing the fractions. Number lines can also be used to support children in finding the difference between fractions. This will be revised later in the block when adding and subtracting fractions.

Encourage children to spot patterns on number lines when simplifying, rather than thinking about fractions individually.

Things to look out for

- Children may find it difficult to place a fraction on the number line when the denominator is greater than the value of the divisions on the number line.
- When crossing 1, children may not be confident in converting mixed numbers/improper fractions.

Key questions

- How many intervals are there on the number line? What is each interval worth?
- What equivalent fractions have you found?
- Is this fraction in its simplest form? How do you know?
- Can you divide the number line into more intervals to place the fractions more accurately?
- How will you place one sixteenth on a number line that is counting in eighths?
- Which fraction was the easiest/hardest to label? Why?

Possible sentence stems

- From my number line, I can see that _____ is equivalent to _____
- When I count in eighths, I can change _____ into _____ because they are equivalent.

National Curriculum links

- Use common factors to simplify fractions; use common multiples to express fractions in the same denomination

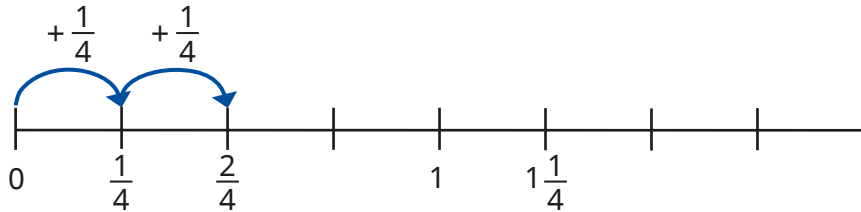
Equivalent fractions on a number line

Key learning

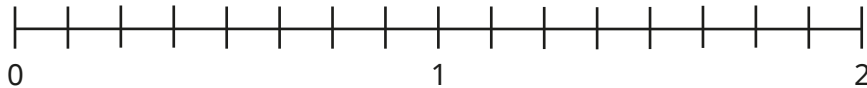
- Jack is counting in quarters.

He writes each number on a number line.

Complete the number line.

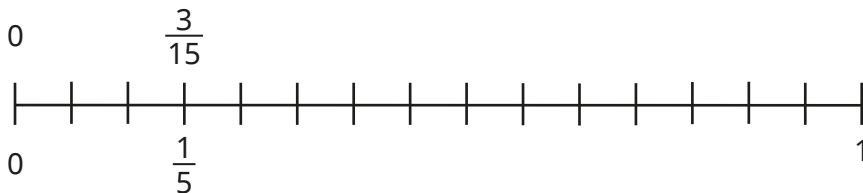


- Use the number line to count forward in eighths.



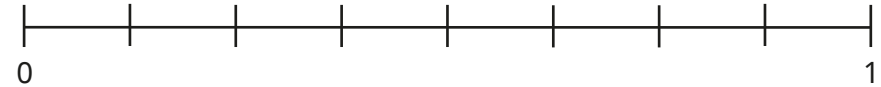
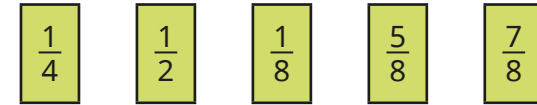
Which of the fractions can be simplified?

- Count in fifteenths on this number line and then write the fractions in their simplest form.



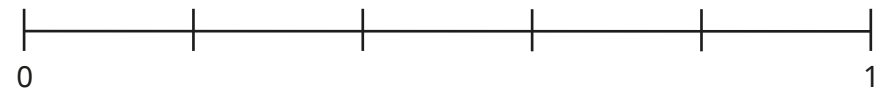
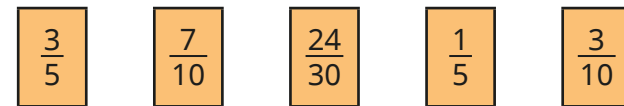
What patterns can you see?

- Label the fractions on the number line.

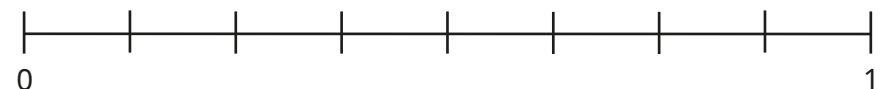
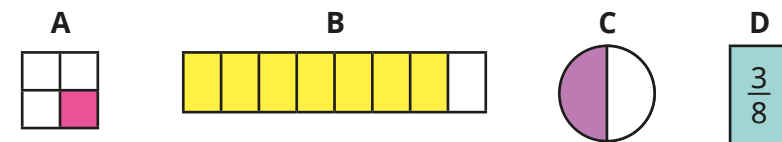


What is the difference between the greatest and smallest fraction?

- Label the fractions on the number line.



- Label A, B, C and D on the number line.



Equivalent fractions on a number line

Reasoning and problem solving

Rosie is counting back in tenths.

She starts at $2\frac{1}{10}$ and counts back 7 tenths.

What number does Rosie end on?

Show this on a number line.

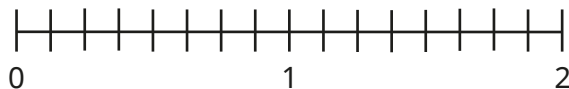
Simplify the fraction.



$$1\frac{4}{10} = \frac{14}{10}$$

$$1\frac{2}{5} = \frac{7}{5}$$

How many ways can you show a difference of one quarter on the number line?



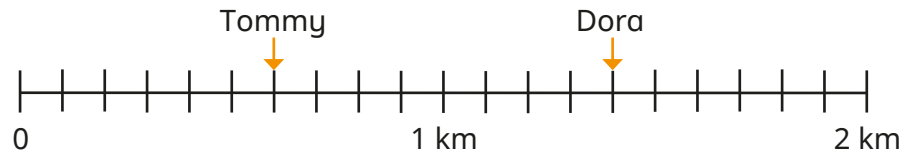
multiple possible answers, e.g.

$$\frac{7}{8} \text{ to } 1\frac{1}{8}$$

$$1\frac{1}{4} \text{ to } 1\frac{1}{2}$$

Dora and Tommy are completing a 2 km race.

The number line shows how far they have run so far.



How much further has Dora run than Tommy?

How much further do they each need to run?

Write your answers in their simplest form.

Huan has completed $\frac{1350}{2000}$ of the race.

Label the number line to show how far Huan has run so far.

$$\frac{4}{5} \text{ km} \quad / \quad \text{Dora: } \frac{3}{5} \text{ km} \quad \text{Tommy: } 1\frac{2}{5} \text{ km}$$

$$\text{halfway between } 1\frac{3}{10} \text{ and } 1\frac{4}{10}$$

Compare and order (denominator)

Notes and guidance

In this small step, children compare and order fractions with the same denominator. Building on the skills covered in the previous steps, they first need to use their knowledge of equivalent fractions to find a common denominator in order to compare.

Children begin by using bar models to help compare fractions. They first work with pairs of fractions where one denominator is a multiple of the other, building on learning from Year 5.

They then look at pairs of fractions where the denominators are not multiples of each other, using their knowledge of multiples and common multiples. Encourage children to find the first common multiple, but allow them to explore different methods. Once children are confident expressing fractions with a common denominator, they use this to order fractions.

Things to look out for

- Some children may compare the numerators without looking at the denominators and finding equivalent fractions.
- Children may not always find the most efficient common multiple when multiplying the denominators, for example expressing $\frac{1}{6}$ and $\frac{2}{9}$ as $\frac{9}{54}$ and $\frac{12}{54}$ rather than $\frac{3}{18}$ and $\frac{4}{18}$

Key questions

- How could you use a number line or a bar model to help you compare the fractions?
- If the denominators are the same, how do you compare the fractions?
- Is one denominator a multiple of the other?
- If one denominator is not a multiple of the other, what do you need to do to be able to compare the fractions?
- How is comparing mixed numbers different from comparing proper fractions? How is it similar?

Possible sentence stems

- I am comparing _____ and _____. I can use _____ as the common denominator.
- If one denominator is not a multiple of the other, I need to find a _____

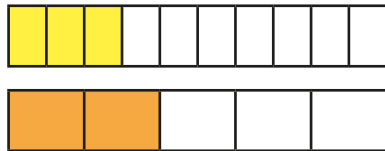
National Curriculum links

- Compare and order fractions, including fractions > 1
- Use common factors to simplify fractions; use common multiples to express fractions in the same denomination

Compare and order (denominator)

Key learning

- The bar models show $\frac{3}{10}$ and $\frac{2}{5}$



Which fraction is greater? How do you know?

- Alex is comparing $\frac{1}{5}$ and $\frac{4}{15}$

She uses equivalent fractions to help.

$$\frac{1}{5} = \frac{3}{15} \quad \frac{3}{15} < \frac{4}{15} \text{ so } \frac{1}{5} < \frac{4}{15}$$

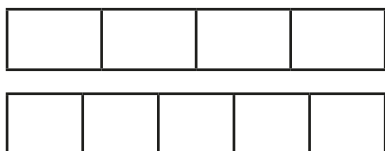
Use Alex's method to compare the fractions.

$$\frac{3}{20} \text{ and } \frac{1}{10}$$

$$\frac{3}{4} \text{ and } \frac{20}{36}$$

$$1\frac{2}{5} \text{ and } 1\frac{10}{30}$$

- Use the bar models to compare $\frac{3}{4}$ and $\frac{2}{5}$



$$\frac{3}{4} \bigcirc \frac{2}{5}$$

- Aisha is comparing $\frac{5}{6}$ and $\frac{3}{4}$ by finding the first common multiple of the denominators.

$$\frac{5}{6} = \frac{10}{12} \quad \frac{3}{4} = \frac{9}{12}$$

$$\frac{10}{12} > \frac{9}{12} \text{ so } \frac{5}{6} > \frac{3}{4}$$

Use Aisha's method to compare the fractions.

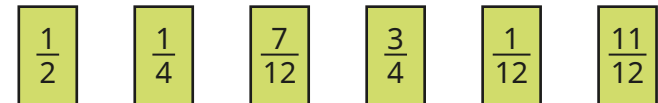
$$\frac{4}{5} \bigcirc \frac{3}{4}$$

$$\frac{3}{5} \bigcirc \frac{4}{7}$$

$$\frac{3}{4} \bigcirc \frac{7}{10}$$

$$2\frac{2}{5} \bigcirc 2\frac{3}{8}$$

- Write the fractions in descending order.



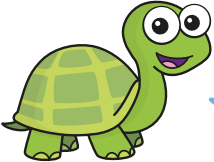
- Write the fractions in ascending order.



Compare and order (denominator)

Reasoning and problem solving

Tiny compares $\frac{3}{5}$ and $\frac{7}{15}$



$\frac{7}{15}$ is greater because 7 is greater than 3

No

Is Tiny correct?
Explain your answer.

Use the digit cards to complete the statements.


5 **3**

$\frac{\square}{4} > \frac{\square}{6}$ $\frac{\square}{4} < \frac{6}{\square}$

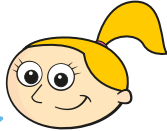
No

$\frac{5}{4} > \frac{3}{6}$
 $\frac{3}{4} < \frac{6}{5}$ or $\frac{5}{4} < \frac{6}{3}$

Eva, Teddy and Amir are reading the same book.




I read $7\frac{3}{4}$ pages in 10 minutes.




Eva

I read $\frac{15}{2}$ pages in 10 minutes.



Amir

I read $3\frac{1}{2}$ pages in 5 minutes.



Teddy

Who is reading the fastest?
How do you know?

Eva



Compare and order (numerator)

Notes and guidance

In the previous small step, children compared and ordered fractions using a common denominator. They now compare and order fractions with the same numerator.

Bar models are a useful representation to explore fractions with the same numerator, starting with unit fractions and then moving on to non-unit fractions. This will lead to the understanding that if the numerators are the same, then the greater the denominator, the smaller the fraction.

Children could visualise or place fractions on a number line and think about whether it is greater than or less than $\frac{1}{2}$ or if it is close to 0 or 1. Understanding can then be built on to compare fractions greater than 1

Children should consider whether it is more efficient to find a common numerator or a common denominator.

Things to look out for

- $\frac{1}{4}$ may be seen as smaller than $\frac{1}{5}$ because 4 is less than 5
- Children may need to be encouraged to use their knowledge of 0, 1 and $\frac{1}{2}$ to help compare fractions, for example $\frac{6}{10} > \frac{2}{7}$ because $\frac{6}{10} > \frac{1}{2}$ and $\frac{2}{7} < \frac{1}{2}$

Key questions

- How can you compare the fractions shown in the bar model?
- Do you need to change one or both numerators? Why?
- Is this fraction closer to 0 or 1?
- Is this fraction greater or less than $\frac{1}{2}$?
- Is it more efficient to find a common numerator or a common denominator?

Possible sentence stems

- When the numerators are the same, the _____ the denominator, the _____ the fraction.
- I know _____ is greater than $\frac{1}{2}$ because ...
- I know _____ is closer to 1 than _____ because ...

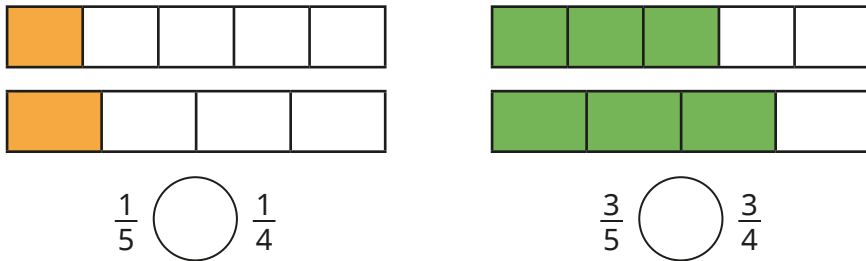
National Curriculum links

- Compare and order fractions, including fractions > 1

Compare and order (numerator)

Key learning

- Write $<$, $>$ or $=$ to compare the fractions.



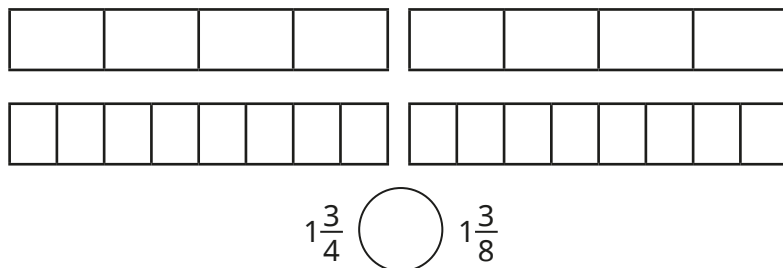
Complete the sentence.

When the numerators are the same, the _____ the denominator, the _____ the fraction.

- Write $<$, $>$ or $=$ to compare the fractions.



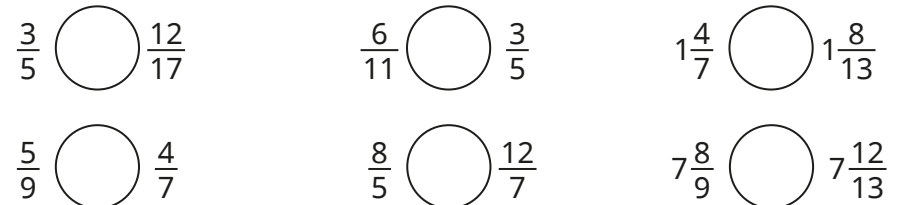
- Complete the bar models and write $<$, $>$ or $=$ to compare the fractions.



- Whitney is comparing $\frac{2}{5}$ and $\frac{6}{13}$ using a common numerator.

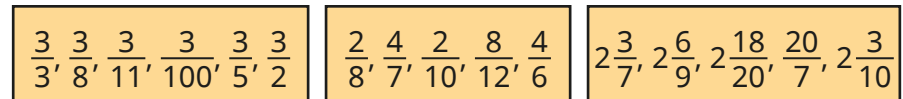
$$\frac{2}{5} = \frac{6}{15} \quad \frac{6}{15} < \frac{6}{13} \quad \text{so} \quad \frac{2}{5} < \frac{6}{13}$$

Use Whitney's method to compare the numbers.



- Dani and Tom have completed a quiz.
Dani answered 7 out of 12 of her questions correctly.
Tom answered 21 out of 30 of his questions correctly.
Who got a greater fraction of their questions correct?

- Write each set of fractions in ascending order.



Compare and order (numerator)

Reasoning and problem solving

Brett is comparing $\frac{3}{7}$ and $\frac{6}{11}$

How many different ways can he work this out?

Find a pair of fractions where it would be more efficient to find:

- a common numerator
- a common denominator.

Compare answers with a partner.



multiple possible answers, e.g. common numerator:

$$\frac{3}{7} = \frac{6}{14}, \frac{6}{14} < \frac{6}{11}$$

$$\text{so } \frac{3}{7} < \frac{6}{11}$$

What could the missing number be, to make the statement true?

$$\frac{1}{5} > \frac{1}{\square} > \frac{1}{12}$$

Is there more than one answer?

How do you know?



6, 7, 8, 9, 10 or 11

Two different pieces of wood have had a fraction of their length chopped off.



Here are the pieces now, showing the fraction that is left.



B

90 cm

Which piece of wood was longer to begin with?

Explain your answer.



The second piece of wood was 1 m long before it was cut.

How long was the first piece of wood?

Add and subtract simple fractions

Notes and guidance

Before beginning, it may be appropriate to revise adding and subtracting fractions with the same denominator to remind children that where the denominators are the same, they need to add/ subtract the numerators and leave the denominator unchanged. In this small step, children build on previous learning in this block and Year 5 to use equivalent fractions to add and subtract fractions where one denominator is a multiple of the other.

Children may be familiar with some common additions and subtractions such as $\frac{1}{2} + \frac{1}{4} = \frac{3}{4}$ and this is a good example on which to build. They start by using bar models before moving on to finding the first common multiple of the denominators.

As the focus is on addition and subtraction of simple fractions, children are not yet required to work with improper fractions and mixed numbers as this will be looked at later in the block.

Things to look out for

- Children may not realise the need to make the denominators equal before adding.
- Children may add both the numerators and the denominators, for example $\frac{1}{2} + \frac{1}{4} = \frac{2}{6}$
- Children may not always simplify their answers.

Key questions

- Do the fractions have the same denominator?
- When are two fractions equivalent?
- How can you find a common denominator?
- How many of the fractions do you need to convert?
- Now the denominators are the same, how do you add/ subtract the fractions?

Possible sentence stems

- Fractions must have the same _____ before they can be added or subtracted.
- The denominator has been multiplied by _____, so to make the equivalent fraction, multiply the numerator by _____
- When fractions have the same _____, to add or subtract them I just _____ the _____

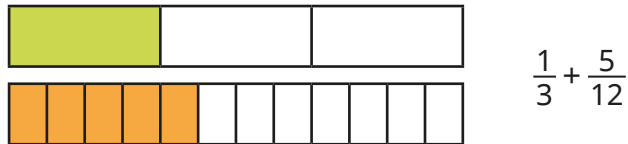
National Curriculum links

- Add and subtract fractions with different denominators and mixed numbers, using the concept of equivalent fractions

Add and subtract simple fractions

Key learning

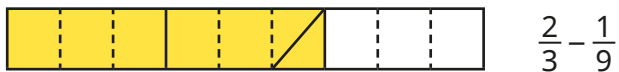
- Use the bar model to help add the fractions.



Work out the additions.

▶ $\frac{1}{3} + \frac{1}{12}$ ▶ $\frac{1}{3} + \frac{7}{12}$ ▶ $\frac{2}{3} + \frac{1}{12}$

- Use the bar model to work out the subtraction.



Work out the subtractions.

▶ $\frac{2}{3} - \frac{2}{9}$ ▶ $\frac{1}{3} - \frac{2}{9}$ ▶ $\frac{2}{3} - \frac{5}{9}$

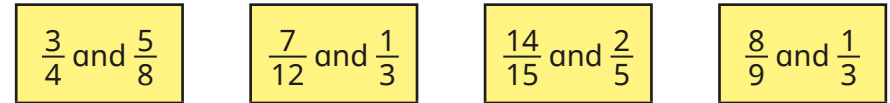
- Here is a method for working out $\frac{7}{10} + \frac{7}{30}$

$$\frac{7}{10} = \frac{21}{30} \quad \frac{21}{30} + \frac{7}{30} = \frac{28}{30} = \frac{14}{15}$$

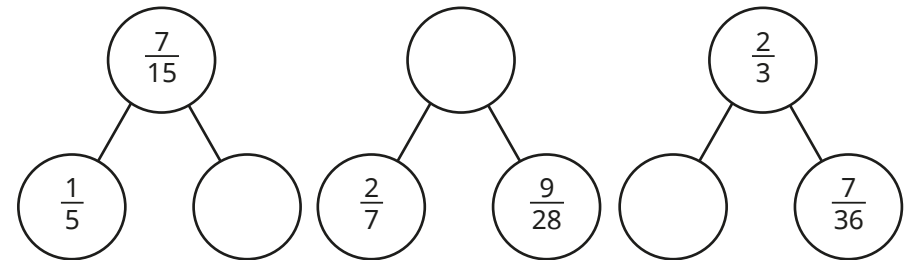
Use this method to work out the additions.

▶ $\frac{2}{9} + \frac{7}{27}$ ▶ $\frac{8}{15} + \frac{1}{5}$ ▶ $\frac{3}{16} + \frac{3}{8} + \frac{1}{4}$

- Find the difference between each pair of fractions.



- Complete the part-whole models.



- Ms Lee has a full tin of paint.
 - She uses $\frac{1}{5}$ of the paint on Monday.
 - She uses $\frac{1}{20}$ on Tuesday.
 - She uses $\frac{3}{10}$ on Wednesday.



How much paint does she have left?

Add and subtract simple fractions

Reasoning and problem solving



Tiny is adding fractions.

Here are Tiny's workings.

$$\frac{3}{5} + \frac{1}{15} = \frac{4}{20} = \frac{1}{5}$$

$$\frac{10}{15} = \frac{2}{3}$$

Explain Tiny's mistake.

Find the correct answer.



Use the same digit in both boxes to complete the calculation.

$$\frac{\square}{20} + \frac{1}{\square} = \frac{9}{20}$$

$$\frac{4}{20} + \frac{1}{4} = \frac{9}{20}$$

$$\frac{5}{20} + \frac{1}{5} = \frac{9}{20}$$

Find all the possible answers.



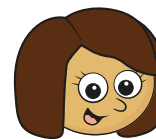
Find the missing number.

$$\frac{3}{5} + \frac{1}{20} = \frac{3}{4} - \frac{\square}{10}$$

1



Kim subtracts $\frac{3}{5}$ from a fraction.



The answer is $\frac{8}{45}$

$\frac{7}{9}$



What fraction has Kim subtracted $\frac{3}{5}$ from?

Give your answer in its simplest form.

Add and subtract any two fractions

Notes and guidance

Following on from the previous small step, children add and subtract fractions where the denominators are not multiples of each other.

Children may need to revisit how to find a common denominator before completing the calculations. They use bar models and then move on to finding the first common multiple of the denominators. Once this is secure, they add up to three fractions or subtract fractions with different denominators.

Children add fractions with answers greater than one, but do not add and subtract mixed numbers until the next step.

Encourage children to simplify answers and convert improper fractions to mixed numbers as appropriate.

Things to look out for

- Children may add both the numerators and the denominators, for example $\frac{1}{3} + \frac{1}{4} = \frac{2}{7}$
- Children may not always simplify their answers.
- Children may leave answers as improper fractions, for example $\frac{7}{5}$

Key questions

- Do the fractions have the same denominator?
- What is the first common multiple of _____ and _____?
- How many of the fractions do you need to convert?
- How do you know if your answer is in its simplest form?
- Do you need to convert your answer to a mixed number? Why or why not?

Possible sentence stems

- The lowest common multiple of _____ and _____ is _____
- To add/subtract the fractions, I could convert them both to _____
- When fractions have the same _____, to add or subtract them you just _____ the _____

National Curriculum links

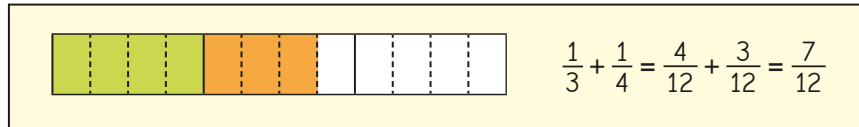
- Add and subtract fractions with different denominators and mixed numbers, using the concept of equivalent fractions
- Identify common factors, common multiples and prime numbers

Add and subtract any two fractions

Key learning

- Esther is working out $\frac{1}{3} + \frac{1}{4}$

She finds a common denominator to work out the answer.



Use Esther's method to work out the additions.

$$\frac{1}{4} + \frac{2}{3}$$

$$\frac{1}{4} + \frac{1}{5}$$

$$\frac{1}{4} + \frac{3}{5}$$

- What common denominator would you use to add each pair of fractions?

$$\frac{2}{4} \text{ and } \frac{1}{5}$$

$$\frac{1}{6} \text{ and } \frac{2}{5}$$

$$\frac{1}{3} \text{ and } \frac{5}{7}$$

$$\frac{3}{8} \text{ and } \frac{4}{7}$$

Find the sum of each pair.

- On Friday, Scott walked $\frac{5}{6}$ km to school, then $\frac{3}{4}$ km to the shop and then $\frac{4}{5}$ km home.

How far did he walk altogether?

- Annie is calculating $\frac{7}{9} - \frac{1}{2}$

She finds the first common multiple of 9 and 2

$$\text{first common multiple of 9 and 2 is 18} \quad \frac{7}{9} - \frac{1}{2} = \frac{14}{18} - \frac{9}{18} = \frac{5}{18}$$

Use this method to find the differences.

$$\frac{2}{3} - \frac{1}{5}$$

$$\frac{4}{9} - \frac{1}{6}$$

$$\frac{5}{7} - \frac{1}{3}$$

$$\frac{11}{12} - \frac{3}{8}$$

- Kim has $\frac{3}{4}$ kg of carrots and $\frac{2}{5}$ kg of potatoes.

She is calculating the total mass of the carrots and potatoes.

$$\frac{3}{4} + \frac{2}{5} = \frac{15}{20} + \frac{8}{20} = \frac{23}{20} = 1\frac{3}{20} \text{ kg}$$

Use Kim's method to find the sums.

Give your answers as mixed numbers.

$$\frac{3}{4} + \frac{3}{5}$$

$$\frac{7}{8} + \frac{1}{3}$$

$$\frac{5}{6} + \frac{5}{7}$$

$$\frac{13}{20} + \frac{2}{3}$$

- Write $<$, $>$ or $=$ to complete the statements.

$$\frac{1}{3} + \frac{1}{5} \bigcirc \frac{4}{5} - \frac{1}{3}$$

$$\frac{1}{3} - \frac{1}{5} \bigcirc \frac{4}{5} - \frac{1}{3}$$

Add and subtract any two fractions

Reasoning and problem solving

Huan and Dora are working out $\frac{1}{4} + \frac{5}{6}$
Here are their methods.

Huan

$$\frac{1}{4} + \frac{5}{6} = \frac{6}{24} + \frac{20}{24} = \frac{26}{24} = 1\frac{2}{24}$$

Dora

$$\frac{1}{4} + \frac{5}{6} = \frac{3}{12} + \frac{10}{12} = \frac{13}{12} = 1\frac{1}{12}$$

Who is correct?

Explain your answer.



Both are correct.

Fill in the boxes to make the calculation correct.

$$1\frac{\square}{10} = \frac{4}{\square} + \frac{\square}{10}$$



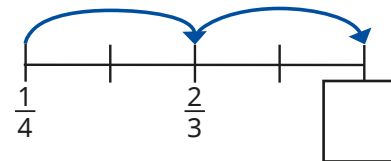
multiple possible answers, e.g.

$$1\frac{1}{10} = \frac{4}{5} + \frac{3}{10}$$

$$1\frac{7}{10} = \frac{4}{5} + \frac{9}{10}$$

The jumps on the number line are equal.

What is the missing value on the number line?



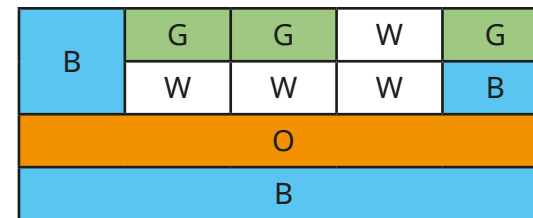
$1\frac{1}{12}$

A wall has been painted in different colours.

$\frac{1}{4}$ of the wall is orange (O).

What fraction of the wall is blue (B)?

What fraction of the wall is white (W)?



blue = $\frac{2}{5}$

white = $\frac{1}{5}$

Add mixed numbers

Notes and guidance

Children encountered mixed numbers in the answers to additions in the previous small step. They now add two mixed numbers, building on their experience of this in Year 5

Children explore adding the wholes and fractional parts separately. This is usually the most efficient method, but converting to improper fractions and then adding is an alternative. Some children may need to revisit converting between improper fractions and mixed numbers.

Questions begin with fractions with the same denominator and then move on to fractions with different denominators. Children can still draw models to represent adding fractions, particularly if these are useful for pairs of fractions with unequal denominators.

Things to look out for

- Children may make errors in the partitioning or recombining of the integer and fractional parts.
- Children may make arithmetical errors when converting to improper fractions with larger numbers.
- The cognitive load is significant when finding solutions to these multi-step problems, so providing scaffolding/ partially started solutions may be useful.

Key questions

- How can you partition the mixed numbers?
- How can the addition be rewritten to make it easier?
- In this question, is it easier to deal with wholes and fractions or to use improper fractions? Why?
- How do you convert a mixed number into an improper fraction?
- Are there any improper fractions in the answer? What can you do about this?

Possible sentence stems

- Mixed numbers can be partitioned into a _____ part and a _____ part.
- A fraction is improper when the _____ is greater than the _____
- _____ is made up of _____ wholes and _____

National Curriculum links

- Add and subtract fractions with different denominators and mixed numbers, using the concept of equivalent fractions
- Identify common factors, common multiples and prime numbers

Add mixed numbers

Key learning

- What method would you use to work out the additions?

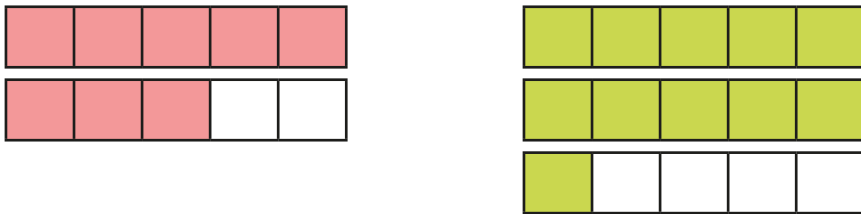
$$3\frac{2}{7} + 4$$

$$3\frac{2}{7} + \frac{4}{7}$$

$$3\frac{2}{7} + 4\frac{4}{7}$$

How are they similar? How are they different?

- Aisha uses a bar model to help work out $1\frac{3}{5} + 2\frac{1}{5} = 3\frac{4}{5}$



Use bar models to help work out the additions.

$$1\frac{2}{7} + 3\frac{2}{7}$$

$$3\frac{2}{7} + 1\frac{4}{7}$$

$$2\frac{1}{7} + 3\frac{5}{7}$$

$$2\frac{1}{7} + 3\frac{6}{7}$$

- Work out the total of each pair of fractions.

$$\frac{3}{11} + \frac{2}{11}$$

$$1\frac{3}{11} + \frac{2}{11}$$

$$1\frac{3}{11} + 1\frac{2}{11}$$

$$2\frac{3}{11} + 1\frac{2}{11}$$

How did you work them out?

Compare methods with a partner.

- Rosie and Amir are working out $1\frac{1}{2} + 2\frac{1}{6}$

Rosie

$$1 + 2 = 3$$

$$\frac{1}{2} + \frac{1}{6} = \frac{3}{6} + \frac{1}{6} = \frac{4}{6}$$

$$3 + \frac{4}{6} = 3\frac{4}{6} = 3\frac{2}{3}$$

Amir

$$1\frac{1}{2} + 2\frac{1}{6} = \frac{3}{2} + \frac{13}{6}$$

$$= \frac{9}{6} + \frac{13}{6}$$

$$= \frac{22}{6} = 3\frac{4}{6} = 3\frac{2}{3}$$

Whose method do you prefer?

Explain your answer.

Use your preferred method to add the mixed numbers.

$$3\frac{1}{2} + 2\frac{3}{8}$$

$$2\frac{1}{9} + 2\frac{2}{5}$$

$$34\frac{1}{9} + 5\frac{5}{8}$$

$$2\frac{3}{4} + 1\frac{4}{5}$$

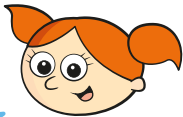
- A jug contains $2\frac{3}{4}$ litres of juice.
Another jug contains $3\frac{3}{5}$ litres of juice.
How much juice is there altogether?





Add mixed numbers

Reasoning and problem solving

Alex, Whitney and Teddy are trying to run 10 km between them.

I ran $2\frac{1}{2}$ km.  Alex

 Whitney I ran $3\frac{4}{5}$ km.

I ran $3\frac{6}{10}$ km.  Teddy

How far have they run?
How much further do they need to run?

$9\frac{9}{10}$ km

$\frac{1}{10}$ km

On Saturday and Sunday, Nijah ran a total of $4\frac{1}{2}$ km. Suggest how far Nijah ran on each day. Find more than one answer.

multiple possible answers, e.g.
 $2\frac{1}{3}$ km and $2\frac{1}{6}$ km

The numbers in the row and column add up to make the totals shown.

$2\frac{1}{4}$	$\square\frac{\square}{8}$	$\frac{1}{2}$	$= 3\frac{7}{8}$
$\frac{1}{\square}$			
$3\frac{1}{12}$			
$= 5\frac{1}{2}$			

Find the missing values.

$1\frac{1}{8}$
 $\frac{1}{6}$

Subtract mixed numbers

Notes and guidance

In this small step, children subtract two mixed numbers, building on the learning from Year 5. Children make links between what is the same and what is different when subtracting mixed numbers compared to adding them.

To introduce this step, children subtract mixed numbers that have the same denominator and do not break the whole. They then subtract fractions with different denominators and complete questions that break the whole. When breaking the whole, children can exchange one whole or convert mixed numbers to improper fractions. Bar models are useful tools to illustrate both methods, and number lines can be used to help find the difference.

Things to look out for

- When breaking the whole, children may be unsure how to exchange.
- Children may make errors when partitioning mixed numbers, for example they may not correctly convert $3\frac{3}{4}$ to $2\frac{7}{4}$
- Children should think about which method is most appropriate for the question, rather than relying on just one method.

Key questions

- How can you partition the mixed number?
- How can the subtraction be rewritten to make it easier?
- In this question, is it easier to deal with wholes and fractions or to use improper fractions? Why?
- How do you convert a mixed number into an improper fraction?

Possible sentence stems

- This calculation will/will not cross the whole because ...
- A fraction is equal to one whole when the _____ is equal to the _____
- The mixed number can be partitioned into _____ and _____
- _____ can be written as _____ wholes and _____

National Curriculum links

- Add and subtract fractions with different denominators and mixed numbers, using the concept of equivalent fractions
- Identify common factors, common multiples and prime numbers

Subtract mixed numbers

Key learning

- What method would you use to work out the subtractions?

$$3\frac{7}{8} - 1$$

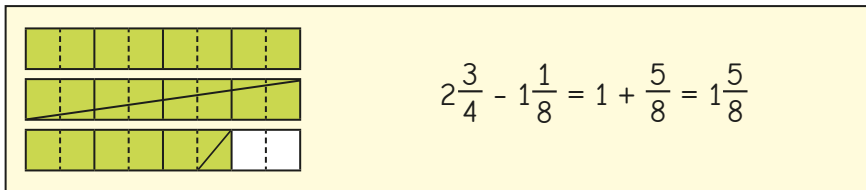
$$3\frac{7}{8} - \frac{3}{8}$$

$$3\frac{7}{8} - 1\frac{3}{8}$$

Compare methods with a partner.

How is this similar to addition? How is it different?

- Tom uses bar models to help work out $2\frac{3}{4} - 1\frac{3}{8}$



Use bar models to help work out the subtractions.

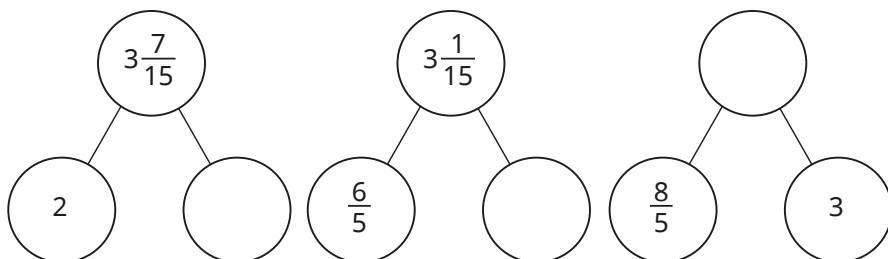
$$2\frac{3}{4} - 1\frac{5}{8}$$

$$3\frac{3}{4} - 2\frac{3}{8}$$

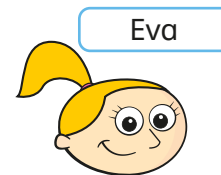
$$2\frac{1}{2} - 1\frac{3}{10}$$

$$4\frac{1}{3} - 2\frac{1}{3}$$

Complete the part-whole models.



- Eva and Tommy are working out $3\frac{3}{5} - 1\frac{7}{10}$



Eva

$$3\frac{3}{5} = 2\frac{8}{5}$$

$$3\frac{3}{5} - 1\frac{7}{10} = 2\frac{8}{5} - 1\frac{7}{10} = 2\frac{16}{10} - 1\frac{7}{10} = 1\frac{9}{10}$$

I cannot subtract the wholes and fractions separately because $\frac{3}{5}$ is less than $\frac{7}{10}$ I will exchange 1 whole for 5 fifths.

I will convert both mixed numbers to improper fractions and then subtract them.



Tommy

$$3\frac{3}{5} - 1\frac{7}{10} = \frac{18}{5} - \frac{17}{10} = \frac{36}{10} - \frac{17}{10} = \frac{19}{10} = 1\frac{9}{10}$$

Choose a method to work out the subtractions.

$$4\frac{4}{5} - 1\frac{9}{10}$$

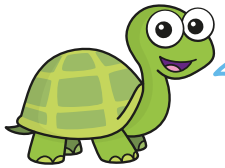
$$2\frac{1}{7} - 1\frac{1}{3}$$

$$3\frac{5}{12} - 1\frac{7}{9}$$

$$3\frac{5}{11} - 1\frac{4}{5}$$


Subtract mixed numbers

Reasoning and problem solving




I cannot work out $3\frac{1}{3} - 1\frac{1}{2}$ because $\frac{1}{2}$ is greater than $\frac{1}{3}$

Is Tiny correct?
Explain your answer.



No

Jack is calculating $4\frac{2}{7} - 2\frac{6}{7}$
He adds $\frac{1}{7}$ to both numbers.

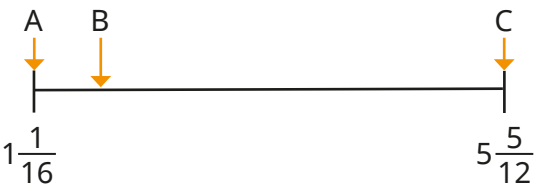


$4\frac{2}{7} - 2\frac{6}{7} = 4\frac{3}{7} - 3$,
so the answer is $1\frac{3}{7}$


Explain why Jack is correct.

He increased both numbers by $\frac{1}{7}$ so the difference remained constant.

On the number line, C is $3\frac{2}{3}$ more than B.



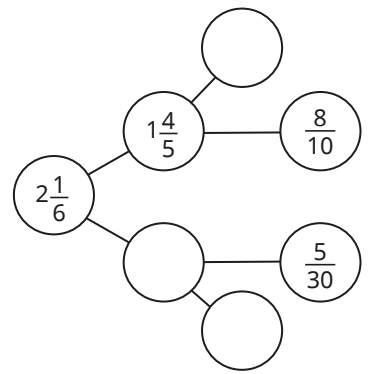
What is the value of B?
What is the difference between A and B?



$1\frac{3}{4}$

$\frac{11}{16}$

Complete the part-whole model.



1

$\frac{11}{30}$

$\frac{1}{5}$

Multi-step problems

Notes and guidance

In this small step, children apply the skills they have learnt in previous steps to solving problems in real-life contexts.

The problems may involve more than one calculation and children need to choose the operations and consider what order to perform them in; this will need careful modelling. Encourage children to think about the most appropriate method to perform any of the calculations. Sharing methods could help children gain a flexible approach to solving the problems.

Children also need to ensure that they write fractions in their simplest form and convert between improper fractions and mixed numbers where appropriate.

Things to look out for

- For longer word problems, the questions may need to be broken down into separate sections to scaffold learning.
- If their understanding is not secure, children may need to revise earlier learning before completing the problems.
- Children may need support to set out solutions with several parts clearly.
- Some children may struggle with the maths because they are overwhelmed by the context of a question.

Key questions

- What can you work out first?
- What do you need to know to work out the answer?
- Can you draw a diagram to represent the problem?
- Can you work out the answer to this part of the problem mentally or do you need another method?
- What can you do next?

Possible sentence stems

- First, I need to work out ...
- The calculation I need to do is ...
- Next, I need to work out ...

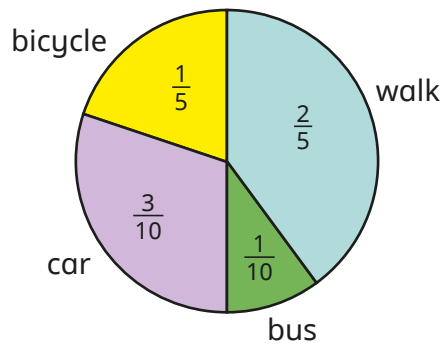
National Curriculum links

- Use common factors to simplify fractions; use common multiples to express fractions in the same denomination
- Add and subtract fractions with different denominators and mixed numbers, using the concept of equivalent fractions
- Solve addition and subtraction multi-step problems in contexts, deciding which operations and methods to use and why
- Solve problems involving addition, subtraction, multiplication and division

Multi-step problems

Key learning

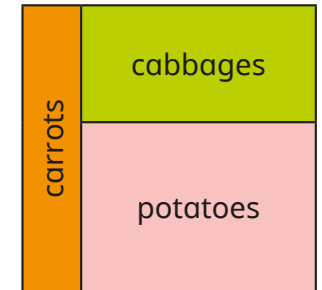
- Children in Class 6 were asked how they travel to school. The results of the survey are shown in the pie chart.



What fraction of children do not get the bus to school?

- Dr Fisher has $\frac{7}{8}$ of a tank of petrol in his car. He drives to see his friend and uses $\frac{1}{5}$ of a tank of petrol. What fraction of a tank of petrol is left in the tank?
- A family buys two equal-sized boxes of cereal. In one week, they eat $\frac{2}{3}$ of box A and $\frac{3}{5}$ of box B. How many boxes of cereal do they eat that week? How many boxes of cereal will they need for three weeks?

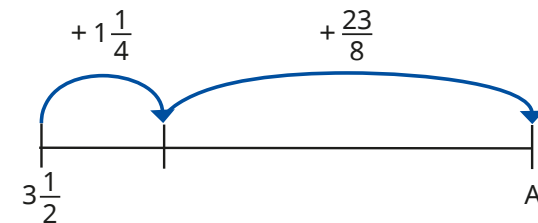
- Here is a vegetable patch. $\frac{1}{5}$ of the patch is for carrots and $\frac{3}{8}$ of the patch is for cabbages. What fraction of the patch is for potatoes?



How much more of the patch is for the potatoes than for the cabbages?

Give all your answers in their simplest form.

- What is the value of A?

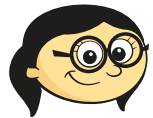


- Whitney has 5 bags of raisins. On Monday, she eats $\frac{2}{3}$ of a bag and gives $\frac{4}{5}$ of a bag away. On Tuesday, she eats $1\frac{1}{3}$ bags and gives $\frac{2}{3}$ of a bag away. How many bags of raisins does Whitney have left?

Multi-step problems

Reasoning and problem solving

Annie and Mo are going on a trip.



Annie

My suitcase has a mass of $29\frac{1}{2}$ kg.

My suitcase is $2\frac{1}{5}$ kg lighter than Annie's.



Mo

What is the total mass of the suitcases?

There is a weight allowance of 32 kg per suitcase.

How much below the weight allowance are Annie and Mo's suitcases?

$$56\frac{4}{5} \text{ kg}$$

$$\text{Annie: } 2\frac{1}{2} \text{ kg}$$

$$\text{Mo: } 4\frac{7}{10} \text{ kg}$$

Find the value of



$$\heartsuit + 3\frac{4}{9} = 6\frac{1}{3}$$

$$8\frac{1}{10} - \heartsuit = \text{Sun}$$

$$\text{Sun} = 5\frac{19}{90}$$

Complete the calculation.

$$2\frac{9}{12} + 3\frac{15}{20} - 2\frac{3}{4} - 2\frac{75}{100} = \boxed{}$$

1

How can you make this calculation simpler?

Autumn Block 4

Fractions B

Small steps

Step 1

Multiply fractions by integers

Step 2

Multiply fractions by fractions

Step 3

Divide a fraction by an integer

Step 4

Divide any fraction by an integer

Step 5

Mixed questions with fractions

Step 6

Fraction of an amount

Step 7

Fraction of an amount – find the whole

Multiply fractions by integers

Notes and guidance

Building on their learning in Year 5, this small step provides practice in multiplying fractions and mixed numbers by integers.

A variety of representations can show that multiplying fractions by integers is the same as repeated addition of a fraction. As when adding and subtracting fractions, the denominator does not change. Children recognise that they need to multiply the numerator by the integer. When multiplying mixed numbers, children can either partition them into wholes and parts, multiplying each of them by the integer, or convert the mixed number to an improper fraction and then multiply the numerator by the integer.

Things to look out for

- Children may multiply both the denominator and numerator by the integer, or only multiply the numerator of the part in a mixed number and not the whole.
- Children may make mistakes when converting between mixed numbers and improper fractions.
- Children should be encouraged to give their answers in their simplest form and convert any improper fractions to mixed numbers.

Key questions

- How is multiplying fractions by integers similar to addition of fractions? How is it different?
- What happens to the denominator when you multiply a fraction by an integer?
- Do you find it easier to partition the mixed number first or to convert it to an improper fraction?
- Is $\frac{2}{3} \times 7$ equal to $7 \times \frac{2}{3}$? Why?

Possible sentence stems

- To multiply a fraction by an integer, I need to multiply the numerator by _____
- To multiply a mixed number by an integer, I can partition it into _____ and _____ and then multiply them both by the integer.
- To multiply a mixed number by an integer, I can convert the mixed number to an _____ and then ...

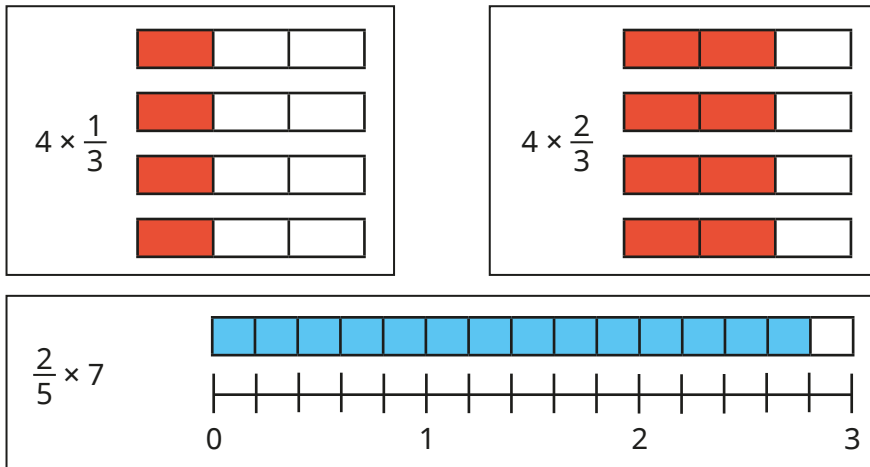
National Curriculum links

- Multiply proper fractions and mixed numbers by whole numbers, supported by materials and diagrams (Y5)

Multiply fractions by integers

Key learning

- Use the diagrams to work out the multiplications.



- Complete the calculations.

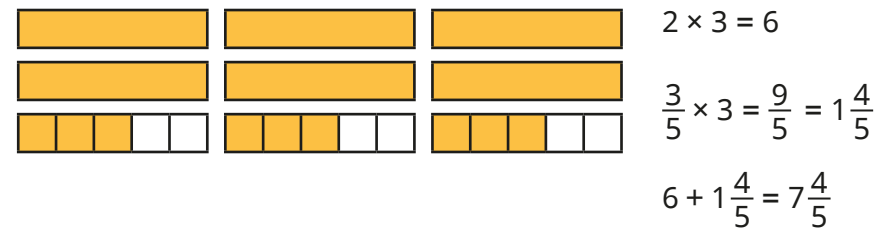
$\frac{3}{5} \times \underline{\quad} = \frac{9}{5} = \underline{\quad}$ $\frac{2}{7} \times \underline{\quad} = \frac{\square}{7} = 1\frac{1}{7}$

- Huan works out $4 \times \frac{7}{8}$

$$4 \times \frac{7}{8} = \frac{28}{8} = 3\frac{4}{8}$$

How can you improve Huan's answer?

- Eva partitions $2\frac{3}{5}$ to help her work out $2\frac{3}{5} \times 3$



Use Eva's method to work out the multiplications.

$2\frac{5}{6} \times 3$

$1\frac{3}{7} \times 5$

$3 \times 2\frac{2}{3}$

$1\frac{1}{6} \times 4$

- Tommy works out $2\frac{3}{5} \times 3$ by converting the mixed number to an improper fraction first.

$$2\frac{3}{5} = \frac{13}{5} \quad \frac{13}{5} \times 3 = \frac{39}{5} \quad \frac{39}{5} = 7\frac{4}{5}$$

Use Tommy's method to work out the multiplications.

$2\frac{2}{5} \times 3$

$1\frac{5}{7} \times 3$

$2 \times 1\frac{3}{4}$

$1\frac{1}{6} \times 2$

Multiply fractions by integers

Reasoning and problem solving

There are 12 children in a class.
The teacher has 4 litres of orange juice.



Each child gets $\frac{1}{5}$ litre of orange juice.
How much orange juice will be left over?

$1\frac{3}{5}$ litres

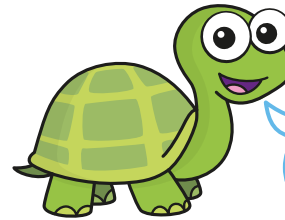
A classroom desk is $1\frac{1}{3}$ m long.
The classroom is 6 m wide.

Will 5 desks fit side by side in the classroom?

Explain your answer.

No
 $5 \times 1\frac{1}{3} = 6\frac{2}{3}$

Tiny is working out $4 \times 3\frac{2}{5}$



The answer
is $12\frac{8}{20}$

Is Tiny correct?
Explain your reasoning.

No

$$A \times 3\frac{1}{5} = B$$

B is an integer.
Work out possible values of A and B.

multiple possible
answers, e.g. A = 5
and B = 16

Multiply fractions by fractions

Notes and guidance

Building on the previous step, children multiply a fraction by another fraction.

Children use concrete and pictorial representations to support them, including folding paper, diagrams and bar models.

By exploring the pictorial representations, children identify the fact that fractions can be multiplied by multiplying both the numerators and denominators. They may need to be reminded that answers should be given in their simplest form.

As the fractions children multiply in this step are all proper, they could be stretched to explain why their answer is always smaller than the fractions given in the question.

Things to look out for

- Children may believe that “multiplication always makes numbers bigger”, but should realise that this is not the case when multiplying by a number less than 1
- The processes for different operations could get mixed up and children may unnecessarily convert to a common denominator as if they are adding or subtracting fractions.

Key questions

- How can you show the calculation as a diagram?
- What is the same and what is different about “half of” a number and “ $\frac{1}{2} \times$ ” a number?
- When you multiply two fractions, is the product greater than or smaller than each of the fractions? Why?
- Why are all of your answers less than 1?

Possible sentence stems

- To show _____, I have split my diagram into _____ equal sections.
- To find the product, I need to ...
- When multiplying a pair of fractions, I need to multiply the _____ and multiply the _____

National Curriculum links

- Multiply simple pairs of proper fractions, writing the answer in its simplest form

Multiply fractions by fractions

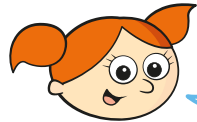
Key learning

- Alex is using a piece of paper to work out $\frac{1}{2} \times \frac{1}{3}$

First, she folds the piece of paper in half.

Then she folds the half into thirds.

Alex shades the fraction that she has created.



I have shaded $\frac{1}{6}$,
so $\frac{1}{2} \times \frac{1}{3} = \frac{1}{6}$

Use Alex's method to work out the multiplications.

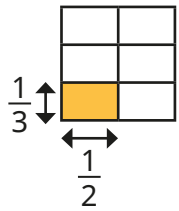
$$\frac{1}{4} \times \frac{1}{2}$$

$$\frac{1}{4} \times \frac{1}{3}$$

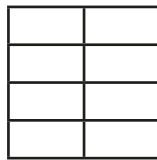
$$\frac{1}{4} \times \frac{1}{4}$$

- Whitney is using diagrams to represent multiplying fractions.

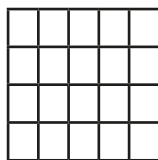
Shade the diagrams to work out the multiplications.



$$\frac{1}{3} \times \frac{1}{2} = \underline{\hspace{2cm}}$$



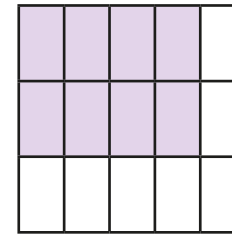
$$\frac{1}{4} \times \frac{1}{2} = \underline{\hspace{2cm}}$$



$$\frac{1}{5} \times \frac{1}{4} = \underline{\hspace{2cm}}$$

Can any of your answers be simplified?

- Dani is using a diagram to work out $\frac{2}{3} \times \frac{4}{5}$



Explain why the diagram shows $\frac{2}{3} \times \frac{4}{5} = \frac{8}{15}$

Use similar diagrams to work out $\frac{2}{3} \times \frac{2}{5}$ and $\frac{2}{3} \times \frac{3}{5}$

- Dexter has spotted a connection between the numerators and the denominators in the question and answer.

$$\frac{3}{4} \times \frac{1}{5} = \frac{3}{20} \quad \frac{4}{5} \times \frac{3}{7} = \frac{12}{35} \quad \frac{4}{5} \times \frac{2}{3} = \frac{8}{15}$$

What connection has Dexter spotted?

Use the connection to work out the multiplications.

$$\frac{2}{5} \times \frac{1}{3}$$

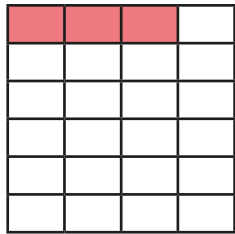
$$\frac{3}{4} \times \frac{3}{5}$$

$$\frac{2}{7} \times \frac{4}{5}$$

Multiply fractions by fractions

Reasoning and problem solving

Aisha uses this diagram to work out the product of two fractions.



What fractions has Aisha multiplied?
What is the answer?

$$\frac{3}{4} \times \frac{1}{6} \text{ or } \frac{1}{6} \times \frac{3}{4}$$

$$\frac{3}{24} = \frac{1}{8}$$

Find the missing numbers.

$$\frac{3}{4} \times \frac{3}{6} = \frac{6}{12}$$

$$= \frac{1}{2}$$

Is there more than one answer?

multiple possible answers, e.g.

$$\frac{2}{3} \times \frac{3}{4} = \frac{6}{12} = \frac{1}{2}$$

$$\frac{2}{6} \times \frac{3}{2} = \frac{6}{12} = \frac{1}{2}$$

Work out the missing numbers.

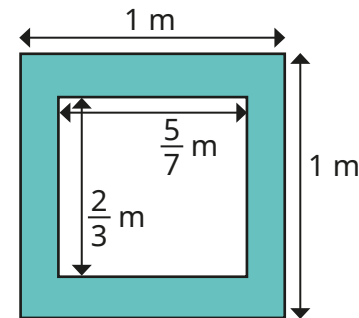
$$\frac{1}{2} \times \frac{1}{\square} = \frac{1}{16}$$

$$\frac{\square}{6} \times \frac{3}{5} = \frac{21}{30}$$

$$\frac{3}{\square} \times \frac{4}{5} = \frac{3}{5}$$

8, 7, 4

What is the area of the shaded region?



$\frac{11}{21} \text{ m}^2$

Divide a fraction by an integer

Notes and guidance

In this small step, children are introduced to dividing fractions by integers for the first time. They focus on dividing fractions where the numerator is a multiple of the integer they are dividing by, for example $\frac{3}{5}$ divided by 3, or $\frac{6}{7}$ divided by 2

Bar models are used initially to represent fractions and to explore how to divide a fraction by an integer. Children complete the number sentence alongside the representation to encourage them to notice that the denominator stays the same and the numerator is divided by the integer. The idea of unitising could be used to support children with dividing fractions by integers. For example, if they know that 6 ones shared between 2 is equal to 3 ones, and 6 eggs shared between 2 is equal to 3 eggs, then 6 sevenths shared between 2 is equal to 3 sevenths. Links can be made to previous representations when multiplying fractions, for example by looking at the equivalence of $\frac{4}{7} \div 2$ and $\frac{4}{7} \times \frac{1}{2}$

Things to look out for

- Children may divide both the numerator and denominator by the integer.
- Children may be tempted to use an abstract procedure, rather than think carefully about what the question is asking.

Key questions

- How could you represent the fraction?
- How could you split the fraction into _____ equal parts?
- What do you notice about the numerators in the question and the answer?
- What do you notice about the denominators in the question and the answer?
- What changes and what stays the same?
- How can you show the division as a bar model?

Possible sentence stems

- If you divide _____ into equal groups, then each group is _____ because _____ \div _____ = _____
- _____ ones divided by _____ is equal to _____ ones, so _____ eighths divided by _____ is equal to _____ eighths.

National Curriculum links

- Divide proper fractions by whole numbers

Divide a fraction by an integer

Key learning

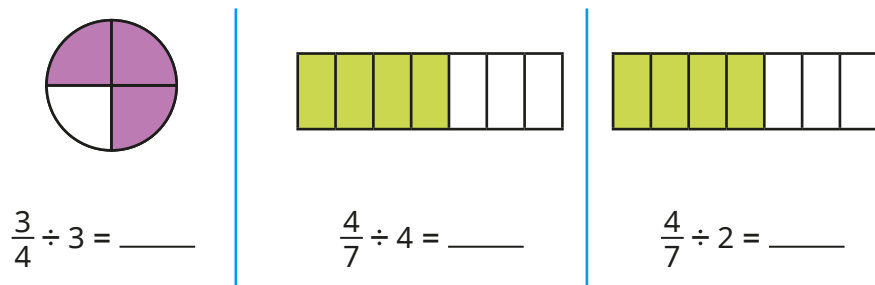
- Filip has $\frac{2}{5}$ of a chocolate bar.

He shares it with his friend.

What fraction of the chocolate bar do they each get?



- Use the diagrams to help you work out the divisions.



- Use the division fact $12 \div 4 = 3$ to work out the divisions.

▶ $12,000 \div 4$ ▶ $12 \text{ m} \div 4$ ▶ $12\text{p} \div 4$ ▶ $\frac{12}{19} \div 4$

- Complete the divisions.

▶ $\frac{6}{11} \div 3$ ▶ $\frac{15}{17} \div 5$ ▶ $\frac{49}{50} \div 7$ ▶ $\frac{96}{101} \div 12$

- A cake has a mass of $\frac{8}{9}$ kg.

- ▶ What is the mass of each piece if the cake is cut into 8 equal pieces?
- ▶ What is the mass of each piece if the cake is cut into 4 equal pieces?
- ▶ What is the mass of each piece if the cake is cut into 2 equal pieces?

- Find the missing integers.

▶ $\frac{15}{16} \div \text{_____} = \frac{5}{16}$

▶ $\frac{20}{23} \div \text{_____} = \frac{4}{23}$

▶ $\frac{15}{16} \div \text{_____} = \frac{3}{16}$

▶ $\frac{20}{23} \div \text{_____} = \frac{5}{23}$

- Mo works out $1\frac{3}{5} \div 2$ using improper fractions.

$$1\frac{3}{5} \div 2 = \frac{8}{5} \div 2 = \frac{4}{5}$$

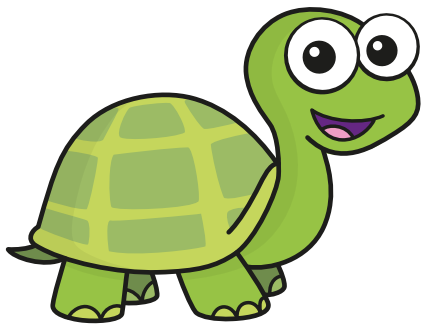
Use Mo's method to work out the divisions.

▶ $1\frac{1}{3} \div 2$ ▶ $1\frac{7}{9} \div 4$ ▶ $2\frac{5}{8} \div 3$ ▶ $3\frac{3}{4} \div 5$

Divide a fraction by an integer

Reasoning and problem solving

Dividing by 2 is the same as finding half of a number, so $\frac{4}{11} \div 2$ is the same as $\frac{1}{2} \times \frac{4}{11}$



Yes

Do you agree with Tiny?

Explain your answer.



Amir and Alex each have a piece of ribbon that is $\frac{99}{100}$ m long.



- Amir cuts his ribbon into 9 equal pieces.
- Alex cuts her ribbon into 3 equal pieces.

Whose pieces of ribbon are longer?

By how much?

Give your answer in centimetres.

Compare methods with a partner.



Alex
22 cm

What could the missing numbers be?



$$\frac{\square}{21} \div 4 = \frac{\square}{21}$$

Can any of your answers be simplified?

multiple possible answers, e.g.

$$\frac{4}{21} \div 4 = \frac{1}{21}$$

$$\frac{12}{21} \div 4 = \frac{3}{21} = \frac{1}{7}$$

Divide any fraction by an integer

Notes and guidance

In this small step, children build on their learning from the previous step to divide fractions where the numerator is not a multiple of the integer they are dividing by.

Children continue to use models and draw diagrams to divide fractions into equal parts. There are two methods that children could use throughout this step. They could use their prior knowledge of equivalent fractions combined with learning from the previous step to find an equivalent fraction where the numerator is a multiple of the integer they are dividing by. Alternatively, through the use of diagrams, children could explore the link between multiplying by a unit fraction and dividing by an integer. When using this method, children should be encouraged to spot the pattern that the numerator stays the same and the denominator is multiplied by the integer.

Encourage children to compare methods and decide which is more efficient, and why.

Things to look out for

- Following on from the previous step, children may try to divide the numerator by the integer anyway even when it is not a multiple, for example $\frac{3}{5} \div 2 = \frac{1.5}{5}$
- Children may become over-reliant on quick tricks.

Key questions

- How can you split a fraction into equal parts? What is each part of the fraction worth?
- How can you show the division as a bar model?
- How is $\frac{1}{3} \div 2$ similar to $\frac{1}{3} \times \frac{1}{2}$?
- What fractions are equivalent to _____?
- Why does finding an equivalent fraction help you to divide a fraction by an integer?
- What multiplication can you use to work out _____ \div _____?

Possible sentence stems

- The bar is split into _____ equal parts.
- I am dividing each _____ by _____, so I must split each part into _____ equal parts.
- _____ is equivalent to _____, so _____ \div _____ is equal to _____ \div _____


National Curriculum links

- Divide proper fractions by whole numbers

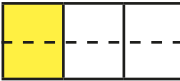
Divide any fraction by an integer

Key learning


- Teddy divides one third into 2 equal parts.




Each part is worth $\frac{1}{6}$
so $\frac{1}{3} \div 2 = \frac{1}{6}$




Draw diagrams to work out the divisions.



- Annie is dividing $\frac{2}{3}$ by 4



I will find an equivalent fraction to help me divide the numerator equally.



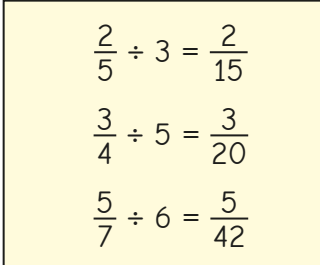

$$\frac{2}{3} = \frac{4}{6}$$

$$\frac{4}{6} \div 4 = \frac{1}{6}$$

Use equivalent fractions to work out the divisions.



- Jack is dividing fractions by integers.

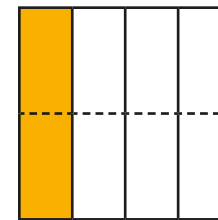
I've noticed something!

What has Jack noticed?

- Work out the divisions.



- Use the diagram to explain why $\frac{1}{4} \div 2 = \frac{1}{4} \times \frac{1}{2} = \frac{1}{8}$



- Work out the missing numbers.

$$\frac{1}{3} \div 2 = \frac{3}{4} \times \frac{\square}{\square} = \frac{\square}{\square}$$

$$\frac{3}{5} \div 2 = \frac{3}{5} \times \frac{\square}{\square} = \frac{\square}{\square}$$

Divide any fraction by an integer

Reasoning and problem solving

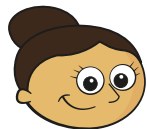
The children are working

out $\frac{6}{7} \div 2$



Ron

$$6 \div 2 = 3, \text{ so } \frac{6}{7} \div 2 = \frac{3}{7}$$



Dora

$$\frac{1}{7} \div 2 = \frac{1}{14}, \text{ so } \frac{6}{7} \div 2 = \frac{6}{14}$$



Whitney

$$\frac{6}{7} \div 2 = \frac{6}{7} \times \frac{1}{2} = \frac{6}{14}$$

Explain why each child is correct.

Discuss each of their methods.

$\frac{3}{7}$ and $\frac{6}{14}$
are equivalent.

Is the statement true or false?

$$\frac{3}{5} \div 4 = \frac{3}{4} \div 5$$

Explain your answer.

True

Find the missing fractions
and integers.

$$\underline{\hspace{2cm}} \div 4 = \frac{7}{36}$$

$$\frac{3}{20} \div \underline{\hspace{2cm}} = \frac{3}{80}$$

$$\underline{\hspace{2cm}} \div \underline{\hspace{2cm}} = \frac{2}{5}$$

Is there more than one possible
answer for each calculation?

$\frac{7}{9}$ or equivalent

4

multiple possible
answers, e.g.

$\frac{4}{5}$ and 2

Mixed questions with fractions

Notes and guidance

Children have now used all four operations with fractions in isolation. In this small step, children identify the appropriate operation(s) to use in a given situation.

Bar models are used to explore word problems and to support children in selecting the correct operation(s). Children start by choosing the correct single operation to solve a problem and move on to explore multi-step problems using all four operations. This step provides a good opportunity to revisit learning from earlier in the year. They can consolidate their knowledge of the order of operations, and also topics such as measure from earlier years.

Things to look out for

- Children may find it difficult to identify the different steps within a problem.
- Children may perform the operations in the wrong order.
- If there are a lot of steps, children may get confused about where they are in the solution to the problem.
- The presence of a fraction in the question may make it feel harder for children, and they could be prompted by considering a similar question with integer values.

Key questions

- Do you need to find the whole or a part? Where can you show this on the bar model?
- What type of calculation do you need to do? How can you tell?
- Does it matter in which order you perform the calculations? Why/why not?
- Which operation should you perform first/second?
- What happens when you insert brackets into the calculation?

Possible sentence stems

- In this calculation, first I need to do _____ and then ...
- To solve the problem, I need to find the _____ of the two fractions.

National Curriculum links

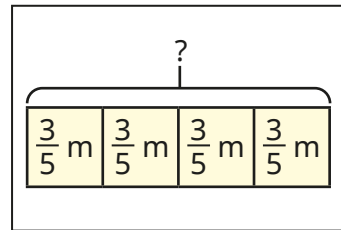
- Add and subtract fractions with different denominators and mixed numbers, using the concept of equivalent fractions
- Multiply simple pairs of proper fractions, writing the answer in its simplest form
- Divide proper fractions by whole numbers
- Solve problems involving addition, subtraction, multiplication and division

Mixed questions with fractions

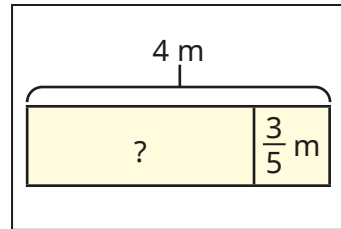
Key learning

- Match the bar models to the correct problems.

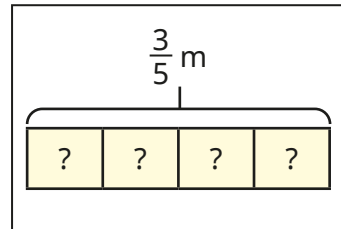
A piece of ribbon is 4 m long. Tom cuts $\frac{3}{5}$ off. How much ribbon is left?



Nijah has 4 pieces of ribbon. Each piece is $\frac{3}{5}$ m long. How much ribbon does Nijah have altogether?

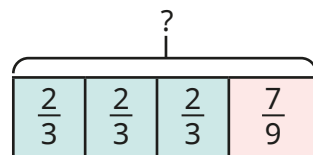


A piece of ribbon is $\frac{3}{5}$ m long. Brett cuts it into 4 equal parts. How long is each part?



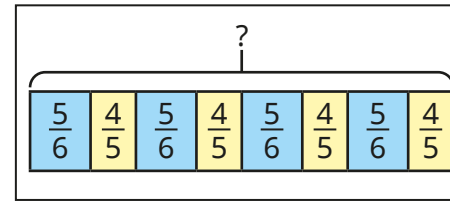
Work out the answer to each problem.

- Find the total length of the bar. Is there more than one way to find the answer?

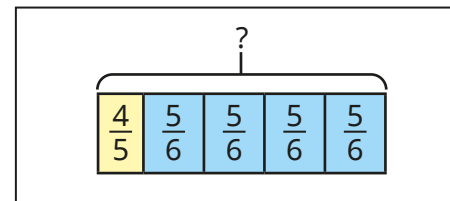


- Find the difference between $\frac{3}{4} \times 3$ and $\frac{3}{4} + 3$

- Match the bar models to the calculations.



$$\left(\frac{4}{5} + \frac{5}{6}\right) \times 4$$



$$\frac{4}{5} + \left(\frac{5}{6} \times 4\right)$$

- Work out the calculations.

$$\triangleright 3\frac{1}{3} + \frac{3}{4} \times 2$$

$$\triangleright 3\frac{1}{3} + \frac{3}{4} \div 2$$

$$\triangleright 3\frac{1}{3} + \frac{3}{4} \div 2$$

- Scott has one-quarter of a bag of sweets.

Kim has two-thirds of a bag of sweets.

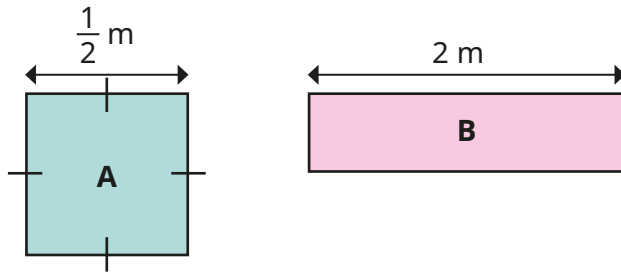
They combine their sweets and share them between themselves and Esther.

What fraction of a bag of sweets does each child get?

Mixed questions with fractions

Reasoning and problem solving

Square A and rectangle B have the same area.
Find the difference between their perimeters.

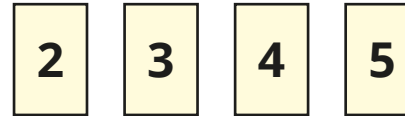


$$2\frac{1}{4} \text{ m}$$

Add two sets of brackets to make the calculation correct.

$$\frac{1}{2} + \frac{1}{4} \times 8 + \frac{1}{6} \div 2 + 1 = 6\frac{1}{18}$$

$$\left(\frac{1}{2} + \frac{1}{4}\right) \times 8 + \frac{1}{6} \div (2 + 1)$$



Using each digit once only, find as many solutions to the calculation that are between 1 and 2 as you can.

$$\frac{\boxed{1}}{\boxed{}} + \boxed{} \times \frac{\boxed{}}{\boxed{}}$$

Compare answers with a partner.

multiple possible answers, e.g.

$$\frac{1}{3} + 2 \times \frac{4}{5}$$

$$\frac{1}{4} + 2 \times \frac{3}{5}$$

$$\frac{1}{5} + 2 \times \frac{3}{4}$$

$$\frac{1}{3} + 4 \times \frac{2}{5}$$

$$\frac{1}{4} + 3 \times \frac{2}{5}$$

$$\frac{1}{5} + 3 \times \frac{2}{4}$$

Fraction of an amount

Notes and guidance

In Year 5, children used bar models to pictorially represent unit and non-unit fractions of an amount. The main focus of this small step is on understanding that the denominator is the number of parts the whole is divided into, and the numerator represents the number of those parts that are selected.

Bar models are a useful way for children to realise the connection between parts and wholes of an amount. By the end of this step, children should be able to find fractions of an amount in different contexts. Encourage them to divide by the denominator and multiply by the numerator, understanding why they are doing this and what they are finding in each step.

Things to look out for

- Children may divide by the numerator instead of the denominator.
- Support may be needed for children who are not fluent with times-tables facts.
- Children may only find the value of the unit fraction and not multiply by the numerator to find the value of the whole fraction.

Key questions

- How do multiplication and division help us when finding fractions of an amount?
- What does dividing the whole amount by the denominator work out?
- How are the parts and wholes represented in a fraction?
- What bar model could you draw to represent the calculation?
- What is the difference between a unit fraction and a non-unit fraction?

Possible sentence stems

- The whole is divided into _____ equal parts. Each part is worth _____
- The numerator is _____, so the fraction is worth _____
- If one fifth is equal to _____, then _____ fifths are equal to _____

National Curriculum links

- Associate a fraction with division and calculate decimal fraction equivalents

Fraction of an amount

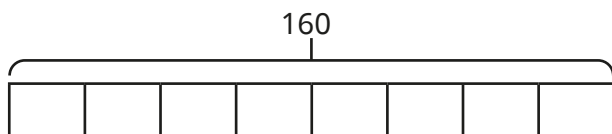
Key learning

- Complete the sentences.
 - ▶ To find one-half of an amount, divide the amount by _____
 - ▶ To find one-third of an amount, divide the amount by _____
 - ▶ To find one-quarter of an amount, divide the amount by _____
 - ▶ To find one-tenth of an amount, divide the amount by _____
 - ▶ To find one-eighteenth of an amount, divide the amount by _____

- Work out the fractions of the amounts.

$\frac{1}{5}$ of 20	$\frac{1}{4}$ of 40	$\frac{1}{5}$ of 30
$\frac{1}{10}$ of £20	$\frac{1}{8}$ of 40 m	$\frac{1}{10}$ of 90 g

- Use the bar model to find the missing numbers.

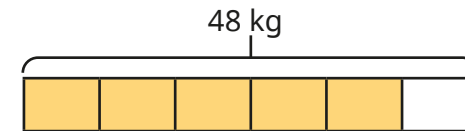


- ▶ $\frac{1}{8}$ of 160 = _____
- ▶ $\frac{5}{8}$ of 160 = _____
- ▶ _____ of 160 = 60

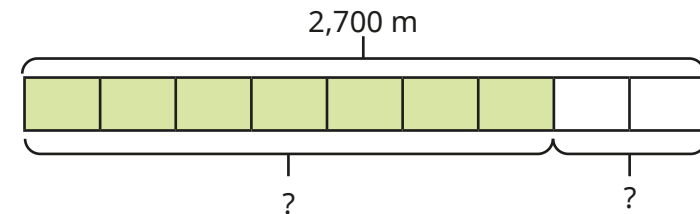
- A cook has 48 kg of potatoes.

He uses $\frac{5}{6}$ of the potatoes.

How many kilograms of the potatoes does he have left?



- Use the bar model to complete the calculations.



- ▶ $\frac{7}{9}$ of 2,700 m = _____ m
- ▶ $\frac{2}{9}$ of 2,700 m = _____ m

- Work out the fractions of the amounts.

$\frac{3}{8}$ of 40	$\frac{5}{6}$ of 18	$\frac{3}{4}$ of 160	$\frac{4}{7}$ of 35
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Fraction of an amount

Reasoning and problem solving

Fill in the missing numbers.

$$\frac{\square}{6} \text{ of } \pounds 300 = \pounds 250$$

5

$$420 \text{ g} = \frac{\square}{12} \text{ of } 720 \text{ g}$$

7

Amir is saving to buy a new game that costs $\pounds 120$

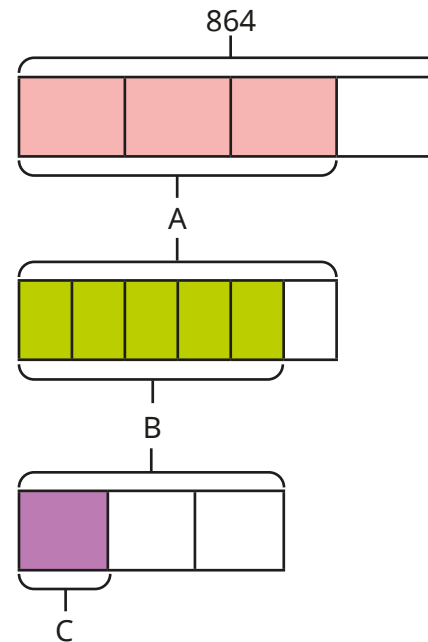
He has saved $\frac{2}{3}$ of the money so far.

How much more money does Amir need to save?

$\pounds 40$

Compare methods with a partner.

Find the values of A, B and C.



$$A = 648$$

$$B = 540$$

$$C = 180$$

Compare methods with a partner.

Fraction of an amount – find the whole

Notes and guidance

In the previous step, children found a fraction of an amount. In this small step, they find the whole amount given a fraction of it.

Using a bar model to represent the parts and the whole is a useful support to children when working through this step. When finding the whole from a unit fraction, a pictorial representation helps children to understand why they simply need to multiply the given amount by the denominator. They then find a unit fraction from a given non-unit fraction and use this to find the whole.

Draw attention to the fact that, when calculating the whole, their answer will be greater than the number in the question. This will help children to sense check their answer.

Fluency with times-tables facts is very helpful here; some children may need a times-table square as support.

Things to look out for

- Children may misinterpret $\frac{3}{4}$ of _____ = 24 as “Find $\frac{3}{4}$ of 24”
- Without pictorial support, children may find it difficult to work out whether to divide or multiply by the numerator/denominator.

Key questions

- How many equal parts are there altogether?
- How many equal parts do you know the value of?
- What is the value of each equal part?
- How can you find the whole?
- Should the whole be greater than or less than the value you are given? Why?

Possible sentence stems

- If one-sixth is equal to _____, then the whole is equal to _____
- If five-sixths is equal to _____, then one-sixth is equal to _____ and the whole is equal to _____
- The whole is split into _____ equal parts.
- To find one part, I need to divide by _____
To find the whole, I need to multiply by _____

National Curriculum links

- Associate a fraction with division and calculate decimal fraction equivalents

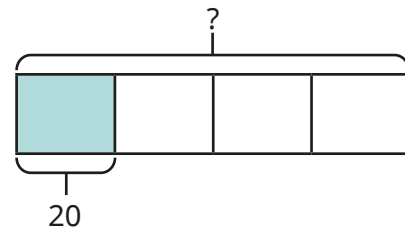
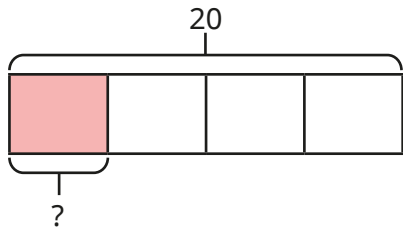
Fraction of an amount – find the whole

Key learning

- Complete the calculations.

$$\frac{1}{4} \text{ of } 20 = \underline{\hspace{2cm}}$$

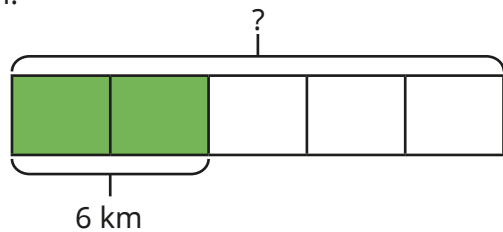
$$\frac{1}{4} \text{ of } \underline{\hspace{2cm}} = 20$$



What is the same about the calculations?

What is different?

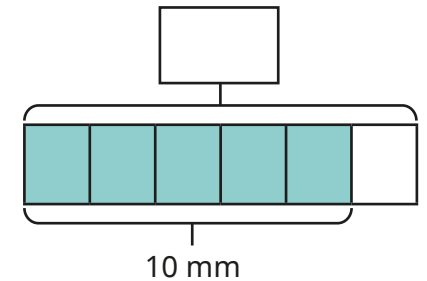
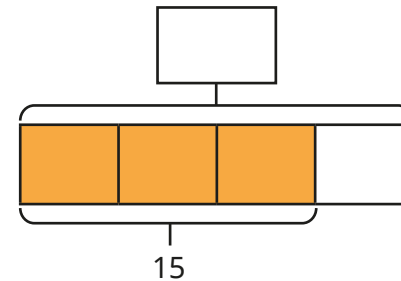
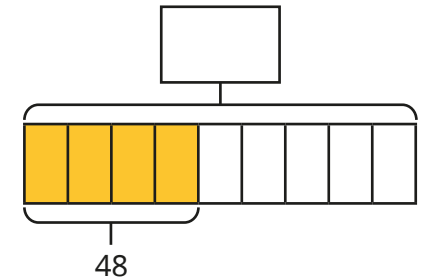
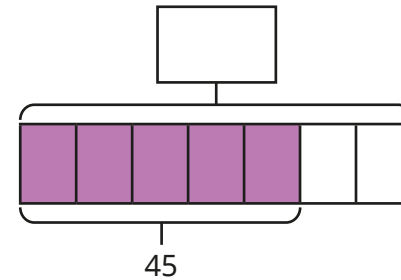
- Tommy runs $\frac{2}{5}$ of a race for his running club.
He runs 6 km.



How far is $\frac{1}{5}$ of the race?

How far is the race altogether?

- Work out the missing wholes.



- Find the missing numbers.

▶ $\frac{3}{5}$ of $\underline{\hspace{2cm}}$ = 21

▶ £180 = $\frac{3}{7}$ of $\underline{\hspace{2cm}}$

▶ $\frac{\square}{3}$ of 60 = 40

▶ $\frac{2}{\square}$ of 80 = 32

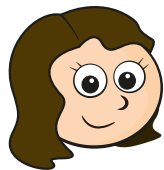
Fraction of an amount – find the whole

Reasoning and problem solving

Miss Rose lights a candle before she has a bath.

After her bath, $\frac{2}{5}$ of the candle is left.

This part of the candle measures 13 cm.



Before my bath, the candle measured 65 cm.

No

Is Miss Rose correct?

Explain your reasoning.

Class 6 voted for their favourite ice cream flavour.

The table shows the fraction of the class that voted for each flavour.

Strawberry	$\frac{1}{4}$
Raspberry	$\frac{1}{6}$
Vanilla	$\frac{1}{12}$
Chocolate	$\frac{3}{8}$
Bubblegum	$\frac{1}{8}$

6 children in the class voted for strawberry.

How many children are there in Class 6?

How many children voted for chocolate?

24

9